Answer on Question #56686 – Math – Combinatorics | Number Theory

3. The coefficient of x^{1502} in the expansion of $\{(1+x+x^2)^{2007}(1-x)^{2008}\}$ is
(a)${}^{2007}C_{501} - {}^{2006}C_{500}$ (b) ${}^{2006}C_{500} - {}^{2006}C_{501}$ (c) ${}^{2007}C_{498} - {}^{2006}C_{499}$
(d) ${}^{2007}C_{501} - {}^{2006}C_{1506}$

Solution. So as

$$(1+x+x^2)^{2007}(1-x)^{2008} = (1+x+x^2)^{2007}(1-x)^{2007}(1-x) = (1-x^3)^{2007}(1-x),$$

$$(1-x^3)^{2007} = \sum_{k=0}^{2007} {}^{2007}C_k\,(-1)^k x^{3k},$$

hold and $1502 = 3 \cdot 500 + 2$, then we have that the coefficient of x^{1502} in the expansion of given expressions is equal to zero. Among all given answers only d ${}^{2007}C_{501} - {}^{2007}C_{1506}$ is zero

Answer: (*d*) ${}^{2007}C_{501} - {}^{2007}C_{1506} = 0.$

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4. *X* and *Y* are any 2 five digits numbers, total number of ways of forming *X* and *Y* with repetition, so that these numbers can be added without using the carrying operation at any stage, is equal to

(a) $45 \cdot 55^4$ (b) $36 \cdot 55^4$ (c) 55^5 (d) $51 \cdot 55^4$

Solution. Total number of ways of a choice for the last figures of numbers *X* and *Y* is equal to

$$1+2+3+\cdots+8+9+10 = \frac{1+10}{2} \cdot 10 = 55$$

It will be similar for the second figures, the third figures and the fourth figures of these numbers since the end. For the first figures we have

$$1+2+3+\cdots+8 = \frac{1+8}{2} \cdot 8 = 36$$

ways of a choice. Thus, total number of ways of forming *X* and *Y* with repetition, so that these numbers can be added without using the carrying operation at any stage, is equal to

$$36 \cdot (55)^4$$

Answer: (*b*): $36 \cdot (55)^4$.
