Answer on Question #56685 – Math – Combinatorics | Number Theory

1. The letters of the word 'GHAJINI' are permuted and all the permutations are arranged in a alphabetical order as in an English dictionary , then total number of words that appear after the word 'GHAJINI' is given by:
(a) 2093 (b) 2009 (c) 2092 (d) 2091

Solution

After the word 'GHAJINI' we must write 'GHAJNII' (1 word). Then we write the words 'GHAN***' ($^3\bar{P}(2) = \frac{3!}{2!} = 3$ words), the words 'GHI****' ($^4P = 4! = 24$ words), the words 'GH$\binom{J}{N}$***' ($2 \cdot {}^4\bar{P}(2) = 2 \cdot \frac{4!}{2!} = 24$ words), the words 'GI*****' ($^5P = 5! = 120$ words), the words 'G$\binom{J}{N}$*****' ($2 \cdot {}^5\bar{P}(2) = 2 \cdot \frac{5!}{2!} = 120$ words), the words '$\begin{pmatrix} H \\ J \\ N \end{pmatrix}$******' ($3 \cdot {}^6\bar{P}(2) = 3 \cdot \frac{6!}{2!} = 3 \cdot 360 = 1080$ words) and the words 'I******' ($^6P = 6! = 720$ words). Thus, total number of words that appear after the word 'GHAJINI' is given by

$$720 + 1080 + 120 + 120 + 24 + 24 + 3 + 1 = 2092$$

Answer: (c): 2092.

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2. If John is allowed to select at most $(n+1)$ chocolates from a collection of $(2n+2)$ distinct chocolates, then total number of ways by which John can select at least two chocolates are given by:
(a) $4^n + 4 \cdot {}^{2n+1}C_n - 2n + 1$ (b) $2 \cdot 4^n + 4 \cdot {}^{2n+1}C_n - 2n + 3$
(c) $2 \cdot 4^n - {}^{2n+1}C_n - 2n - 3$ (d) $2 \cdot 4^n + {}^{2n+1}C_n - 2n - 3$

Solution

Total number of ways by which John can select at least 2 and at most $(n+1)$ chocolates are given by

$$^{2n+2}C_2 + {}^{2n+2}C_3 + \cdots + {}^{2n+2}C_{n+1}.$$

Let it be equal to x. So as

$$^{2n+2}C_2 + {}^{2n+2}C_3 + \cdots + {}^{2n+2}C_{n+1} = {}^{2n+2}C_{2n} + {}^{2n+2}C_{2n-1} + \cdots + {}^{2n+2}C_{n+1}$$

and

$$^{2n+2}C_0 + {}^{2n+2}C_1 + \cdots + {}^{2n+2}C_{2n+1} + {}^{2n+2}C_{2n+2} = 2^{2n+2},$$

hold, then

$$2^{2n+2} = 2x + 2 \cdot ({}^{2n+2}C_0 + {}^{2n+2}C_1) - {}^{2n+2}C_{n+1}$$

and $x = 2^{2n+1} - ({}^{2n+2}C_0 + {}^{2n+2}C_1) + \frac{1}{2}\,{}^{2n+2}C_{n+1} = 2 \cdot 4^n - 1 - 2n - 2 + {}^{2n+1}C_{n+1} =$
$= 2 \cdot 4^n - 2n - 3 + {}^{2n+1}C_{n+1}.$

Answer: (d) $2(4)^n + {}^{2n+1}C_{n+1} - 2n - 3$.
