Answer on Question #56674 – Math – Real Analysis

Let A be non empty subset of real numbers which is bounded above. Let -A be the set of all real numbers –x, where -x belong A. Show that sup(A)= -inf(-A)

Solution

A is a non empty subset of real number which is bounded above, so sup (A) exists. By definition, for all $x \in A$, we have that $\sup(A) \geq x$, so $-\sup(A) \leq -x$ for all $x \in A$.

- A is non empty subset of real numbers which is bounded below, so inf (-A) exists. So we have that $\inf(-A) \leq y$ for all $y \in -A$, then $-\inf(-A) \geq y$ for all $y \in -A$, so $-\inf(-A) \geq x$ for all $x \in A$. $-\inf(-A)$ is a upper bound of *A*.

By definition, *x* is a supremum if *x* is an upper bound of *A* and for any other upper bound *y* of *A* $x \leq y$. From these definitions and from what was proven above it is clear that $-\inf(A) \geq \sup(-A)$ and $-\sup(A) \leq \inf(-A)$. Combining these two inequalities gives sup(A)= -inf(-A), as required.