Answer on Question #56670 – Math – Real Analysis

If x and y are irrational numbers, show that $x + y$ and $x * y$ are irrational.

Solution

This statement is not necessarily true.

Let's consider some of irrational numbers. For instance

$$\sqrt{2};\quad 2\sqrt{2};\ 2-\sqrt{3};\qquad 2+\sqrt{3}$$

Now, find the sum and the multiplication of these numbers.

$$\sqrt{2}+2\sqrt{2}=3\sqrt{2}$$

In this example a sum of two irrational is irrational.

But:

$$\left(2-\sqrt{3}\right)+\left(2+\sqrt{3}\right)=4$$

In this case sum of two irrationals is rational.

The same situation for the multiplication of two irrational numbers. For example

$$\sqrt{2}*2\sqrt{2}=4$$

Four is rational

But for multiplication of $\sqrt{2}$ and $2+\sqrt{3}$ we have

$$\sqrt{2}*\left(2+\sqrt{3}\right)=2\sqrt{2}+\sqrt{6}$$

It is irrational.