

Answer on Question #56665 – Math – Algebra

1. What is the value of the fourth term in a geometric sequence for which $a^1 = 15$ and $r = 1/3$
Express your answer as a fraction.

Solution

$$a_1 = 15; r = \frac{1}{3}$$

$$\text{Because } a_2 = a_1 \cdot r; a_3 = a_2 \cdot r; a_4 = a_3 \cdot r,$$

then

$$\begin{aligned} a_4 &= a_3 \cdot r = (a_2 \cdot r) \cdot r = a_2 r^2 = (a_1 \cdot r)r^2 = a_1 \cdot r^3 = 15 \cdot \left(\frac{1}{3}\right)^3 = \\ &= 15 \cdot \frac{1}{81} = \frac{15}{81} = \frac{5}{27}. \end{aligned}$$

Answer: $\frac{5}{27}$.

2. The second term in a geometric sequence is 12. The fourth term in the same sequence is $4/3$.
What is the common ratio in this sequence?

Solution

$$a_2 = 12; a_4 = \frac{4}{3}$$

$$\text{Because } a_4 = a_3 \cdot r; a_3 = a_2 \cdot r, \text{ then } a_4 = a_3 \cdot r = (a_2 \cdot r) \cdot r = a_2 \cdot r^2.$$

$$\text{Thus, } r^2 = \frac{a_4}{a_2},$$

$$r^2 = \frac{4}{3} : 12 = \frac{1}{9},$$

$$r = \pm \sqrt{\frac{1}{9}} = \pm \frac{1}{3}.$$

Answer: $\frac{1}{3}$ or $-\frac{1}{3}$.

3. What is the sum of the first five terms of a geometric series with $a^1 = 20$ and $r = \frac{1}{4}$.
Express your answer as an improper fraction in lowest terms without using spaces.

Solution

$$a_1 = 20; r = \frac{1}{4}$$

$$S_5 = a_1 + a_2 + a_3 + a_4 + a_5 = a_1 + a_1 \cdot r + a_2 \cdot r + a_3 \cdot r + a_4 \cdot r =$$

$$= a_1 + a_1 \cdot r + a_1 \cdot r^2 + a_1 \cdot r^3 + a_1 \cdot r^4.$$

$$\text{Or } S_5 = \frac{a_1 - a_5 \cdot r}{1 - r}.$$

$$\text{So, } S_5 = \frac{20 - a_5 \cdot \frac{1}{4}}{1 - \frac{1}{4}}$$

$$a_5 = a_1 \cdot r^4 = 20 \cdot \left(\frac{1}{4}\right)^4 = \frac{20}{256} = \frac{5}{64}.$$

$$\text{Thus, } S_5 = \frac{20 - \frac{5}{64} \cdot \frac{1}{4}}{1 - \frac{1}{4}} = \frac{20 - \frac{5}{256}}{\frac{3}{4}} = \frac{5115 \cdot 4}{256 \cdot 3} = \frac{1705}{64}.$$

$$\text{Answer: } \frac{1705}{64}.$$