QUESTION C

If $(C_0+C_1)(C_1+C_2)(C_2+C_3)\cdots(C_{n-1}+C_n)=m*C_1C_2C_3\cdots C_{n-1}$, then m is equal to

SOLUTION

$$\boxed{C_i=\frac{n!}{i!(n-i)!}}$$

$$(C_0+C_1)(C_1+C_2)(C_2+C_3)\cdots(C_{n-1}+C_n)=m*C_1C_2C_3\cdots C_{n-1}$$

$$\boxed{m=\frac{(C_0+C_1)(C_1+C_2)(C_2+C_3)\cdots(C_{n-1}+C_n)}{C_1C_2C_3\cdots C_{n-1}}}$$

$$(C_0+C_1)(C_1+C_2)(C_2+C_3)\cdots(C_{n-1}+C_n)=$$

$$=\left(\frac{n!}{0!(n-0)!}+\frac{n!}{1!(n-1)!}\right)\cdots\left(\frac{n!}{(n-1)!(n-(n-1))!}+\frac{n!}{n!(n-n)!}\right)=$$

$$=\left(\frac{n!}{0!(n-0)!}+\frac{n!}{1!(n-1)!}\right)\cdots\left(\frac{n!}{(n-1)!(n-(n-1))!}+\frac{n!}{n!(n-n)!}\right)=$$

$$=n!\left(\frac{1}{1!(n)!}+\frac{n}{1!n!}\right)\cdots n!\left(\frac{n}{n!1!}+\frac{1}{n!1!}\right)=(n!)^n\frac{(n+1)^n}{1!^2 2!^2\cdots n!^2}$$

$$\boxed{(C_0+C_1)(C_1+C_2)(C_2+C_3)\cdots(C_{n-1}+C_n)=(n!)^n\frac{(n+1)^n}{1!^2 2!^2\cdots n!^2}}$$

$$C_1C_2C_3\cdots C_{n-1}=\frac{n!}{1!(n-1)!}\frac{n!}{2!(n-2)!}\cdots\frac{n!}{(n-1)!(n-(n-1))!}=$$

$$=\frac{\overbrace{n!*n!*n!*n!\cdots n!}^{n-1}}{1!^2 2!^2\cdots(n-1)!^2}=\frac{n!^{n-1}}{1!^2 2!^2\cdots(n-1)!^2}$$

$$\boxed{C_1C_2C_3\cdots C_{n-1}=\frac{n!^{n-1}}{1!^2 2!^2\cdots(n-1)!^2}}$$

$$m=\frac{(C_0+C_1)(C_1+C_2)(C_2+C_3)\cdots(C_{n-1}+C_n)}{C_1C_2C_3\cdots C_{n-1}}=\frac{(n!)^n\frac{(n+1)^n}{1!^2 2!^2\cdots n!^2}}{\frac{n!^{n-1}}{1!^2 2!^2\cdots(n-1)!^2}}=$$

$$=\frac{(n!)^n(n+1)^n 1!^2 2!^2\cdots(n-1)!^2}{n!^{n-1}1!^2 2!^2\cdots(n-1)!^2 n!^2}=\frac{n!^n(n+1)^n}{n!^{n-1}*n!^2}=\frac{(n+1)^n}{n!}$$

ANSWER

$$\boxed{m = \frac{(n+1)^n}{n!}}$$

ANSWER (p)

QUESTION D

If C_r are the binomial co-efficients in the expansion of $(1+x)^n$, the value of

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(i+j)C_iC_j$$

is

SOLUTION

We need the following properties of binomial coefficients

$$\boxed{C_i = \frac{n!}{i!(n-i)!}}$$

$$C_0 = \frac{n!}{0!*(n-0)!} = \frac{n!}{n!} = 1$$

$$(1+x)^n|_{x=1} = 2^n = \left(\sum_{i=0}^{n} C_i * 1^i * x^{n-i}\right)\Bigg|_{x=1} = \sum_{i=0}^{n} C_i * 1^i * 1^{n-i} \Longleftrightarrow \sum_{i=0}^{n} C_i = 2^n$$

$$\sum_{i=0}^{n} C_i = C_0 + \sum_{i=1}^{n} C_i = 1 + \sum_{i=1}^{n} C_i = 2^n \Longleftrightarrow \boxed{\sum_{i=1}^{n} C_i = 2^n - 1}$$

$$\sum_{i=1}^{n} i * C_i = \sum_{i=1}^{n} i * \frac{n!}{i!*(n-i)!} = \sum_{i=1}^{n} \frac{n!}{(i-1)!*(n-i)!} = \left(\begin{array}{l} i-1=t \\ i=1 \rightarrow t=0 \\ i=n \rightarrow t=n-1 \end{array}\right) =$$

$$= \sum_{t=0}^{n-1} \frac{n!}{t!*(n-t-1)!} = \sum_{t=0}^{n-1} \frac{n*(n-1)!}{t!*((n-1)-t)!} = n * \sum_{t=0}^{n-1} \frac{(n-1)!}{t!*((n-1)-t)!} =$$

$$= n * \sum_{t=0}^{n-1} C_t = n * 2^{n-1}$$

$$\boxed{\sum_{i=1}^{n} i * C_i = n * \sum_{t=0}^{n-1} C_t = n * 2^{n-1}}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}(i+j)C_iC_j = \sum_{i=1}^{n}\sum_{j=1}^{n} i*C_iC_j+\sum_{i=1}^{n}\sum_{j=1}^{n} j*C_iC_j = \sum_{i=1}^{n} i*C_i \sum_{j=1}^{n} C_j+\sum_{i=1}^{n} C_i \sum_{j=1}^{n} j*C_j =$$

$$= \underbrace{\sum_{i=1}^{n} i * C_i}_{n*2^{n-1}} \underbrace{\sum_{j=1}^{n} C_j}_{2^n-1} + \underbrace{\sum_{i=1}^{n} C_i}_{2^n-1} \underbrace{\sum_{j=1}^{n} j * C_j}_{n*2^{n-1}} = n*2^{n-1}(2^n-1)+n*2^{n-1}(2^n-1) = n*2^n*(2^n-1)$$

ANSWER

$$\boxed{\sum_{i=1}^{n}\sum_{j=1}^{n}(i+j)C_iC_j = n*2^n*(2^n-1)}$$

ANSWER (q)