Answer on Question #56642 – Math – Algebra

A furniture company is faced with the following the price-demand function, revenue function, and cost function:

$$p(x) = 90 - 5x$$

$$R(x) = xp(x)$$

$$C(x) = 250 + 15x$$

Where p(x) is the price in dollars at which x hundred chairs can be sold and R(x) and C(x) are in thousands of dollars.

a. Give the revenue R for producing 1200 chairs.

b. Find the production level that gives the break-even point.

c. Find the production level that gives the maximum revenue and the maximum profit.

Solution

a. We know that the revenue function in accordance with the condition of the task is equal to

$$R(x) = xp(x)$$

and

$$p(x) = 90 - 5x$$

Thus, we can substitute for p(x) into the revenue formula:

$$R(x) = x(90 - 5x) = 90x - 5x^2$$

Finally, we can determine the revenue value for producing 1200 chairs:

$$R(12) = 90 \cdot 12 - 5 \cdot 12^2 = 1080 - 720 = \$360$$

We can conclude that, the revenue R for producing 1200 chairs is equal to $360,000

b.

The break-even point is the sales level at which total revenue generated is equal to the total cost (fixed and variable), meaning that at the break-even point of a company, the profit is zero.

Thus, the break-even point will be when: $\mathrm{p(x)} = 0$

$$\mathrm{P(x)} = \mathrm{xp(x)} - \mathrm{C(x)}$$

Based on the noted above information, we can write the following:

$$\mathrm{x(90 - 5x) - (250 + 15x) = 0}$$

Now, we can simplify the obtained equation; combine like terms on the left and right sides:

$$\mathrm{90x - 5x^2 - 250 - 15x = 0}$$

$$\mathrm{-5x^2 + 75x - 250 = 0}$$

Then, we can use the factoring in order to find the value of x; divide all terms by 5 and multiply by -1:

$$\mathrm{(x^2 - 15x + 50) = 0}$$

$$\mathrm{(x^2 - 10x - 5x + 50) = 0}$$

$$\mathrm{(x(x - 5) - 10(x - 5))) = 0}$$

$$\mathrm{((x - 5)(x - 10)) = 0}$$

Thus, we consider $\mathrm{x - 5 = 0}$ and $\mathrm{x - 10 = 0, x = 5, x = 10}$

We can conclude that, the production level that gives the break-even point will be equal to 500 chairs and 1000 chairs.

c.

In given problem, we need to determine the production level that gives the maximum revenue and the maximum profit.

In this case, we have to consider the revenue function and subtract the cost function, then take the derivative:

$$\mathrm{P(x) = 90x - 5x^2 - (250 + 15x) = 0}$$

$$\mathrm{P'(x)} = \left(\mathrm{90x - 5x^2 - (250 + 15x)}\right)' = \mathrm{(-5x^2 + 75x - 250)' = -10x + 75}$$

Now, we solve for x:

$$\mathrm{-10x + 75 = 0}$$

$$\mathrm{-10x = -75}$$

$$x = 7.5$$

Thus, the furniture company should produce 750 chairs in order to obtain the maximum profit.

Then, we substitute this value into the profit function and get the following result:

$$P(7.5) = -5(7.5)^2 + 75(7.5) - 250 = -5(56.25) + 562.5 - 250$$
$$= -281.25 + 562.5 - 250 = 31.25$$

The profit will be equal to $31250.

The maximum revenue occurs when production level is

$$R(x) = 90x - 5x^2$$

$$R'(x) = 90 - 10x$$

$$R''(x) = -10 \text{ which is } < 0 \text{ (maximum)}$$

Then, $R'(x_{max}) = 0$

$$90 - 10x_{max} = 0$$

$$10x_{max} = 90$$

$$x_{max} = 9$$

Thus, we can conclude that, the maximum revenue may be achieved by producing 900 chairs and the maximum profit may be achieved by producing 750 chairs.