Answer on Question #56611 – Math – Combinatorics | Number Theory

The sum of the coefficients in the expansion $\left(1-2x+5x^2\right)^n$ *is 'a' and sum of the coefficients in the expansion* $\left(1+x\right)^{2n}$ *is 'b'. Then*

(A) *a=b* (B) $a=b^2$ (C) $a^2=b$ (D) *ab=1*

Solution

Method 1

Prove by mathematical induction that *a=b*.

1) n=1:

$\left(1-2x+5x^2\right)^1$: *a*=4

$\left(1+x\right)^2=1+2x+x^2$: *b*=4.

a=*b*=4*:* it's true.

2) Assume that for n=k it is true. Let the sum of the coefficients in the expansion $\left(1-2x+5x^2\right)^n$ be 'a1' and sum of the coefficients in the expansion be 'b1'.

a1=b1 is true.

3) n=k+1:

$\left(1-2x+5x^2\right)^{k+1}=\left(1-2x+5x^2\right)^k\left(1-2x+5x^2\right)$.

Then *a*=a1*(1-2+5)=4a1.

$\left(1+x\right)^{2(k+1)}=\left(1+x\right)^{2k}\left(1+x\right)^2$.

Then *b*=b1*(1+1)^2=4b1=4a1=*a*.

By mathematical induction, we proved that *a*=*b*.

Answer: *a*=*b*.

Method 2

$$\left(1-2x+5x^2\right)^n=\sum_{k+l=m=0}^{n}\frac{n!}{k!l!m!}1^k\left(-2x\right)^l\left(5x^2\right)^m,$$

$$\left(1+x\right)^{2n}=\sum_{k=0}^{2n}\frac{(2n)!}{k!\left(2n-k\right)!}1^k x^{2n-k}.$$

To find the sum of coefficients in each expansion, put $x=1$ in both formulas.

If the sum of the coefficients in the expansion $\left(1-2x+5x^2\right)^n$ is *a* and the sum of the coefficients in the expansion $\left(1+x\right)^{2n}$ is *b,* then $\left(1-2+5\right)^n=4^n=a$ and $(1+1)^{2n}=2^{2n}=4^n=b$. Thus, $a=b$.

Answer: $a=b$.