Answer on Question #56512 – Math – Statistics and Probability

Question

Paul, Joe and Ken are playing soccer. The probability that Paul scores a goal is ¼, that of Joe scoring is 3/5 and that Ken scoring a goal is 4/7. Find the probability that in a soccer game:

i. Only two scores a goal
ii. Two of them score a goal
iii. None of them score a goal
iv. At least one of them scores a goal

Solution

Let
A="Paul scores a goal", $\bar{A}$= "Paul does not score a goal",
B="Joe scores a goal", $\bar{B}$= "Joe does not score a goal",
C="Ken scores a goal", $\bar{C}$= "Ken does not score a goal".
It is given that $P(A)=\frac{1}{4}$, $P(B)=\frac{3}{5}$, $P(C)=\frac{4}{7}$, hence
$P(\bar{A})=1-P(A)=1-\frac{1}{4}=\frac{3}{4}$, $P(\bar{B})=1-P(B)=1-\frac{3}{5}=\frac{2}{5}$, $P(\bar{C})=1-P(C)=1-\frac{4}{7}=\frac{3}{7}$.
We can use product rule to fill the next table.

Paul scores a goal	Joe scores a goal	Ken scores a goal	Probability
Yes $\left(\frac{1}{4}\right)$	Yes $\left(\frac{3}{5}\right)$	Yes $\left(\frac{4}{7}\right)$	$P(ABC)=\frac{1}{4}\cdot\frac{3}{5}\cdot\frac{4}{7}=\frac{12}{140}$
Yes $\left(\frac{1}{4}\right)$	Yes $\left(\frac{3}{5}\right)$	No $\left(\frac{3}{7}\right)$	$P(AB\bar{C})=\frac{1}{4}\cdot\frac{3}{5}\cdot\frac{3}{7}=\frac{9}{140}$
Yes $\left(\frac{1}{4}\right)$	No $\left(\frac{2}{5}\right)$	Yes $\left(\frac{4}{7}\right)$	$P(A\bar{B}C)=\frac{1}{4}\cdot\frac{2}{5}\cdot\frac{4}{7}=\frac{8}{140}$
Yes $\left(\frac{1}{4}\right)$	No $\left(\frac{2}{5}\right)$	No $\left(\frac{3}{7}\right)$	$P(A\bar{B}\bar{C})=\frac{1}{4}\cdot\frac{2}{5}\cdot\frac{3}{7}=\frac{6}{140}$
No $\left(\frac{3}{4}\right)$	Yes $\left(\frac{3}{5}\right)$	Yes $\left(\frac{4}{7}\right)$	$P(\bar{A}BC)=\frac{3}{4}\cdot\frac{3}{5}\cdot\frac{4}{7}=\frac{36}{140}$
No $\left(\frac{3}{4}\right)$	Yes $\left(\frac{3}{5}\right)$	No $\left(\frac{3}{7}\right)$	$P(\bar{A}B\bar{C})=\frac{3}{4}\cdot\frac{3}{5}\cdot\frac{3}{7}=\frac{27}{140}$
No $\left(\frac{3}{4}\right)$	No $\left(\frac{2}{5}\right)$	Yes $\left(\frac{4}{7}\right)$	$P(\bar{A}\bar{B}C)=\frac{3}{4}\cdot\frac{2}{5}\cdot\frac{4}{7}=\frac{24}{140}$
No $\left(\frac{3}{4}\right)$	No $\left(\frac{2}{5}\right)$	No $\left(\frac{3}{7}\right)$	$P(\bar{A}\bar{B}\bar{C})=\frac{3}{4}\cdot\frac{2}{5}\cdot\frac{3}{7}=\frac{18}{140}$

i. The probability that only two score ball equals
$$P(AB\bar{C})+P(\bar{A}BC)+P(A\bar{B}C)=\frac{9}{140}+\frac{36}{140}+\frac{8}{140}\approx 0.3786.$$

ii. The probability that two of them score ball equals
$$P(AB\bar{C})+P(\bar{A}BC)+P(A\bar{B}C)+P(ABC)=\frac{9}{140}+\frac{36}{140}+\frac{8}{140}+\frac{12}{140}\approx 0.4643.$$

iii. The probability that none of them score ball equals
$$P(\bar{A}\bar{B}\bar{C})=\frac{18}{140}\approx 0.1286.$$

iv. The probability that at least one of them scores ball equals
$$P(ABC)+P(AB\bar{C})+P(A\bar{B}C)+P(A\bar{B}\bar{C})+P(\bar{A}BC)+P(\bar{A}B\bar{C})+P(\bar{A}\bar{B}C)=$$
$$=1-P(\bar{A}\bar{B}\bar{C})=1-\frac{18}{140}\approx 0.8714.$$