**Answer on Question # 56499 – Math – Trigonometry**

1. *If AB is a chord of contact of point P(5, -5) to the circle* $x^2 + y^2 = 5$ *and* $(\alpha, \beta)$ *is the orthocenter of ΔPAB, then* $|\alpha - 1| + |\beta - 1|$ *is equal to*

   (A) 2 (B) $\sqrt{5}$ (C) $2\sqrt{5}$ (D) 3

## Solution

The radius of the circle is $\sqrt{5}$.

Define the coordinates of the point tangent to the circle. Radius is the diagonal of a square: $2a^2 = 5, a = \sqrt{\frac{5}{2}}$

The point tangent to the circle $A\left(-\sqrt{\frac{5}{2}}, -\sqrt{\frac{5}{2}}\right)$.

The point tangent to the circle $B\left(\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$.

The equation of the line $AP$ : $\frac{x-5}{-\sqrt{\frac{5}{2}}-5} = \frac{y+5}{-\sqrt{\frac{5}{2}}+5}$; $\left(5-\sqrt{\frac{5}{2}}\right)x + \left(5+\sqrt{\frac{5}{2}}\right)y = -10\sqrt{\frac{5}{2}}$.

Find the equation of height drawn from vertex B:

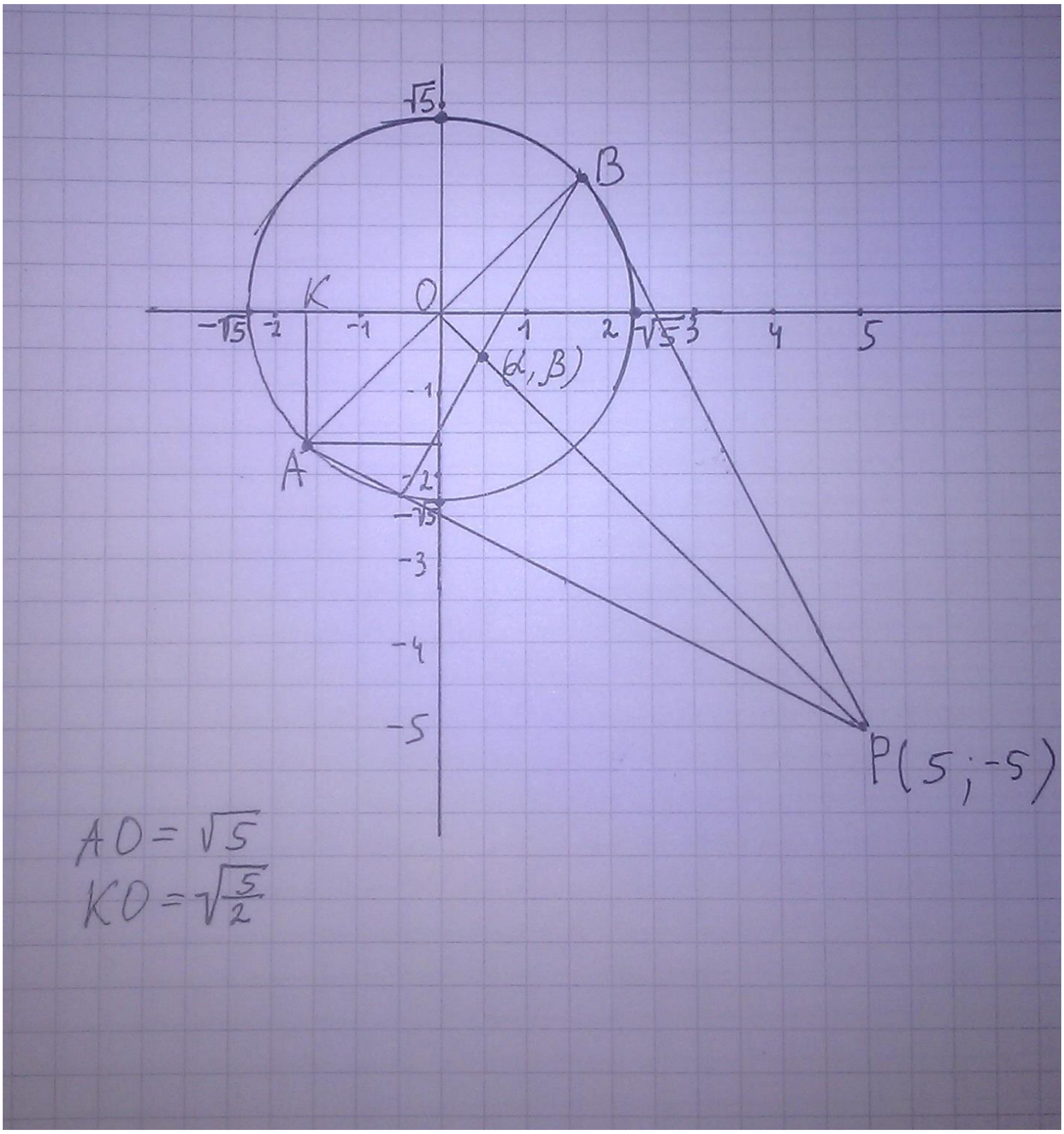
$$\frac{x-\sqrt{\frac{5}{2}}}{5-\sqrt{\frac{5}{2}}} = \frac{y-\sqrt{\frac{5}{2}}}{5+\sqrt{\frac{5}{2}}}; \left(5+\sqrt{\frac{5}{2}}\right)x + \left(\sqrt{\frac{5}{2}}-5\right)y = 5$$

Find the equation of height drawn from vertex P: $y = -x$.

Orthocentre is the point of intersection of heights:

$$\begin{cases} \left(5+\sqrt{\frac{5}{2}}\right)x + \left(\sqrt{\frac{5}{2}}-5\right)y = 5 \\ y = -x \end{cases}$$

Orthocentre is $(\alpha;\beta) = \left(\frac{1}{2}, -\frac{1}{2}\right)$. Then $|\alpha-1| + |\beta-1| = \left|\frac{1}{2}-1\right| + \left|-\frac{1}{2}-1\right| = \frac{1}{2} + \frac{3}{2} = 2$




**Answer:** A) 2.

**2.** $tan^4\frac{\pi}{16} + cot^4\frac{\pi}{16} + tan^4\frac{2\pi}{16} + cot^4\frac{2\pi}{16} + tan^4\frac{3\pi}{16} + cot^4\frac{3\pi}{16}$ is equal to

(A) 645 (B) 646 (C) 848 (D) 678

**Solution**

$$tan^4\frac{\pi}{16}+cot^4\frac{\pi}{16}+tan^4\frac{2\pi}{16}+cot^4\frac{2\pi}{16}+tan^4\frac{3\pi}{16}+cot^4\frac{3\pi}{16}=$$

$$=\frac{sin^4\frac{\pi}{16}}{cos^4\frac{\pi}{16}}+\frac{cos^4\frac{\pi}{16}}{sin^4\frac{\pi}{16}}+\frac{sin^4\frac{\pi}{8}}{cos^4\frac{\pi}{8}}+\frac{cos^4\frac{\pi}{8}}{sin^4\frac{\pi}{8}}+\frac{sin^4\frac{3\pi}{16}}{cos^4\frac{3\pi}{16}}+\frac{cos^4\frac{3\pi}{16}}{sin^4\frac{3\pi}{16}}=$$

$$=\frac{\left(\frac{1-cos\frac{\pi}{8}}{2}\right)^2}{\left(\frac{1+cos\frac{\pi}{8}}{2}\right)^2}+\frac{\left(\frac{1+cos\frac{\pi}{8}}{2}\right)^2}{\left(\frac{1-cos\frac{\pi}{8}}{2}\right)^2}+\frac{\left(\frac{1-cos\frac{\pi}{4}}{2}\right)^2}{\left(\frac{1+cos\frac{\pi}{4}}{2}\right)^2}+\frac{\left(\frac{1+cos\frac{\pi}{4}}{2}\right)^2}{\left(\frac{1-cos\frac{\pi}{4}}{2}\right)^2}+$$

$$+\frac{\left(\frac{1-cos\frac{3\pi}{8}}{2}\right)^2}{\left(\frac{1+cos\frac{3\pi}{8}}{2}\right)^2}+\frac{\left(\frac{1+cos\frac{3\pi}{8}}{2}\right)^2}{\left(\frac{1+cos\frac{3\pi}{8}}{2}\right)^2}=\frac{\left(\frac{1-\sqrt{\frac{1+cos\frac{\pi}{4}}{2}}}{2}\right)^2}{\left(\frac{1+\sqrt{\frac{1+cos\frac{\pi}{4}}{2}}}{2}\right)^2}+$$

$$+\frac{\left(\frac{1+\sqrt{\frac{1+cos\frac{\pi}{4}}{2}}}{2}\right)^2}{\left(\frac{1-\sqrt{\frac{1+cos\frac{\pi}{4}}{2}}}{2}\right)^2}+\frac{\left(\frac{1-\frac{\sqrt{2}}{2}}{2}\right)^2}{\left(\frac{1+\frac{\sqrt{2}}{2}}{2}\right)^2}+\frac{\left(\frac{1+\frac{\sqrt{2}}{2}}{2}\right)^2}{\left(\frac{1-\frac{\sqrt{2}}{2}}{2}\right)}+\frac{\left(\frac{1-\sqrt{\frac{1+cos\frac{3\pi}{4}}{2}}}{2}\right)^2}{\left(\frac{1+\sqrt{\frac{1+cos\frac{3\pi}{4}}{2}}}{2}\right)^2}+$$

$$+\frac{\left(\frac{1+\sqrt{\frac{1+cos\frac{3\pi}{4}}{2}}}{2}\right)^2}{\left(\frac{1-\sqrt{\frac{1+cos\frac{3\pi}{4}}{2}}}{2}\right)^2}=\frac{\left(\frac{1-\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}}{2}\right)^2}{\left(\frac{1+\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}}{2}\right)^2}+\frac{\left(\frac{1+\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}}{2}\right)^2}{\left(\frac{1-\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}}{2}\right)^2}+\frac{\left(\frac{1-\frac{\sqrt{2}}{2}}{2}\right)^2}{\left(\frac{1+\frac{\sqrt{2}}{2}}{2}\right)^2}+\frac{\left(\frac{1+\frac{\sqrt{2}}{2}}{2}\right)^2}{\left(\frac{1-\frac{\sqrt{2}}{2}}{2}\right)}+$$

$$+\frac{\left(\frac{1-\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}}{2}\right)^2}{\left(\frac{1+\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}}{2}\right)^2}+\frac{\left(\frac{1+\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}}{2}\right)^2}{\left(\frac{1-\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}}{2}\right)^2}=\frac{\left(2-\sqrt{2+\sqrt{2}}\right)^2}{\left(2+\sqrt{2+\sqrt{2}}\right)^2}+\frac{\left(2+\sqrt{2+\sqrt{2}}\right)^2}{\left(2-\sqrt{2+\sqrt{2}}\right)^2}+$$

$$+\frac{\left(2-\sqrt{2}\right)^2}{\left(2+\sqrt{2}\right)^2}+\frac{\left(2+\sqrt{2}\right)^2}{\left(2-\sqrt{2}\right)^2}+\frac{\left(2+\sqrt{2-\sqrt{2}}\right)^2}{\left(2-\sqrt{2-\sqrt{2}}\right)^2}+\frac{\left(2-\sqrt{2-\sqrt{2}}\right)^2}{\left(2+\sqrt{2-\sqrt{2}}\right)^2}=322+224\sqrt{2}+322-$$

$$-322+224\sqrt{2}+34=678$$

**Answer:** D) 678.

**3.** If $f(x)=\tan x+\tan^2 x\cdot\tan 2x$ and $g(n)=\sum_{m=0}^{n} f\left(2^m\right)$, then $g(2013)-\tan\left(2^{2014}\right)$ is equal to

(A)0 (B) tan 1 (C) –tan 1 (D) None of these

**Solution**

$$\sum_{n=0}^{2013} tan\left(2^n\right)=tan1+tan2+tan4+\ldots+tan2013;$$

$$\sum_{n=0}^{2013} tan^2\left(2^n\right)tan\left(2\cdot 2^n\right)=tan^2 1tan2+tan^2 2tan4+tan^2 4tan8+\ldots+tan^2 2^{2013}tan2^{2014},$$

$$\sum_{n=0}^{2013}\tan\left(2^n\right)+\sum_{n=0}^{2013}\tan^2\left(2^n\right)\tan\left(2\cdot 2^n\right)=$$

$$=\tan 1(1+\tan 1\tan 2)+\tan 2(1+\tan 2\tan 4)+\tan 4(1+\tan 4\tan 8)+\ldots$$
$$+\tan 2^{2013}\left(1+\tan 2^{2013}\tan 2^{2014}\right)=\tan(2-1)(1+\tan 1\tan 2)+\tan(4-2)(1+\tan 2\tan 4)+$$
$$+\tan(8-4)(1+\tan 4\tan 8)+\ldots+\tan\left(2^{2014}-2^{2013}\right)\left(1+\tan 2^{2013}\tan 2^{2014}\right)$$

Using

$$tan(\alpha-\beta)(1+tan\,\alpha\, tan\,\beta)=tan\,\alpha-tan\,\beta$$

we obtain

$$\sum_{n=0}^{2013}\tan(2^n)+\sum_{n=0}^{2013}\tan^2(2^n)\tan(2\cdot 2^n)=$$

$$=\tan 2-\tan 1+\tan 4-\tan 2+...+\tan 2^{2014}-\tan 2^{2013}=-\tan 1+\tan 2^{2014}$$

After substracting $tan(2^{2014})$ we obtain $-tan1$.

**Answer:** C) $-tan1$.

4. The number of real roots of $\sqrt{5x^2-6x-6}-\sqrt{5x^2-6x-7}=1$ is / are

(A)2 (B) 4 (C) 8 (D) None of these

**Solution**

$$\sqrt{5x^2-6x-6}-\sqrt{5x^2-6x-7}=1;$$

$$\sqrt{5x^2-6x-6}=1+\sqrt{5x^2-6x-7};$$

$$5x^2-6x-6=1+2\sqrt{5x^2-6x-7}+5x^2-6x-7;$$

$$\sqrt{5x^2-6x-7}=0;$$

$$5x^2-6x-7=0;$$

$$x=\frac{3+2\sqrt{11}}{5};\ x=\frac{3-2\sqrt{11}}{5}.$$

**Answer:** A) 2.

5. Let $A$ be average of numbers $2\sin 2^\circ$, $4\sin 4^\circ$, $6\sin 6^\circ, \ldots$, $180\sin 180^\circ$, then $A$ is equal to

(A) $\cot 1^\circ$ (B) $\tan 1^\circ$ (C) $\sin 1^\circ$ (D) $\cos 1^\circ$

**Solution**

$$\sin 180^\circ=0,\ \sin(\alpha)=\sin(180^\circ-\alpha),\ \sin 90^\circ=1.$$

$$x = 2\sin 2^\circ + 4\sin 4^\circ + \ldots + 90\sin 90^\circ + \ldots + 178\sin 178^\circ =$$
$$= (2+178)\sin 2^\circ + (4+176)\sin 4^\circ + \ldots = 180(\sin 2^\circ + \sin 4^\circ + \ldots + \sin 88^\circ) + 90\sin 90^\circ$$

Then

$$\bar{x} = \frac{x}{90} = 2\sin 2^\circ + 2\sin 4^\circ + \ldots + 2\sin 88^\circ + 1;$$

$$\bar{x}\sin 1^\circ = 2\sin 2^\circ \sin 1^\circ + 2\sin 4^\circ \sin 1^\circ + \ldots + 2\sin 88^\circ \sin 1^\circ + \sin 1^\circ;$$

Now

$$2\sin 2^\circ \sin 1^\circ = \cos 1^\circ - \cos 3^\circ;$$

$$2\sin 4^\circ \sin 1^\circ = \cos 3^\circ - \cos 5^\circ;$$

$$2\sin 88^\circ \sin 1^\circ = \cos 87^\circ - \cos 89^\circ.$$

Hence

$$\bar{x}\sin 1^\circ = \cos 1^\circ - \cos 89^\circ + \sin 1^\circ = \cos 1^\circ.$$

Thus

$$\bar{x} = \cot 1^\circ.$$

**Answer:** A) $\cot 1^\circ$.