QUESTION №81

If $\sum\limits_{r=1}^{\infty}\frac{8}{(2r-1)\sqrt{(2r+3)(2r+5)}+(2r+3)\sqrt{(2r-1)(2r+1)}}=\sqrt{a}+\sqrt{\frac{5}{3}}-\sqrt{c}$ where a,c are coprime numbers , then value of $a+\frac{c}{4}$ is equal to

(1) 3 (2) 5 (3) 8 (4) 4

SOLUTION

transform denominator of expression

$$(2r-1)\sqrt{(2r+3)(2r+5)}+(2r+3)\sqrt{(2r-1)(2r+1)}$$

Note that all expressions are symmetric with respect to the expression $2r+2$. We introduce replacement $t=2r+2$

$$(2r-1)\sqrt{(2r+3)(2r+5)}+(2r+3)\sqrt{(2r-1)(2r+1)}=$$

$$=(2r+2-3)\sqrt{(2r+2+1)(2r+2+3)}+(2r+2+1)\sqrt{(2r+2-3)(2r+2-1)}=$$

$$=(t-3)\sqrt{(t+1)(t+3)}+(t+1)\sqrt{(t-3)(t-1)}=$$

$$=\sqrt{t-3}\sqrt{t+1}(\sqrt{(t-3)*(t+3)}+\sqrt{(t+1)*(t-1)})=$$

$$=\sqrt{t-3}\sqrt{t+1}(\sqrt{t^2-9}+\sqrt{t^2-1})$$

$$\sum_{r=1}^{\infty}\frac{8}{(2r-1)\sqrt{(2r+3)(2r+5)}+(2r+3)\sqrt{(2r-1)(2r+1)}}=$$

$$=\sum_{t}\frac{8}{\sqrt{t-3}\sqrt{t+1}(\sqrt{t^2-9}+\sqrt{t^2-1})}=\sum_{t}\frac{8(\sqrt{t^2-9}-\sqrt{t^2-1})}{\sqrt{t-3}\sqrt{t+1}(t^2-9-t^2+1)}=$$

$$=\sum_{t}\frac{8(\sqrt{t^2-9}-\sqrt{t^2-1})}{-8\sqrt{t-3}\sqrt{t+1}}=-\sum_{t}\frac{\sqrt{t^2-9}-\sqrt{t^2-1}}{\sqrt{t-3}\sqrt{t+1}}=$$

$$=-\sum_{t}\left(\sqrt{\frac{t+3}{t+1}}-\sqrt{\frac{t-1}{t-3}}\right)=\sum_{t}\left(\sqrt{\frac{t-1}{t-3}}-\sqrt{\frac{t+3}{t+1}}\right)=$$

$$=\sum_{r=1}^{\infty}\left(\sqrt{\frac{2r+2-1}{2r+2-3}}-\sqrt{\frac{2r+2+3}{2r+2+1}}\right)=\sum_{r=1}^{\infty}\left(\sqrt{\frac{2r+1}{2r-1}}-\sqrt{\frac{2r+5}{2r+3}}\right)=$$

$$=\sqrt{3}+\sqrt{\frac{5}{3}}+\sqrt{\frac{7}{5}}+\sqrt{\frac{9}{7}}+\cdots-\sqrt{\frac{7}{5}}-\sqrt{\frac{9}{7}}-\cdots=\sqrt{3}+\sqrt{\frac{5}{3}}$$

$$a = 3 \qquad c = 0 \longrightarrow a + \frac{c}{4} = 3$$

ANSWER

$$(1)\ 3$$

QUESTION №82

The equation of circle which touches axis of y at the origin and passes though (3,4) is

$$(1)\ \ 2(x^2 + y^2) - 3x = 0$$
$$(2)\ \ 3(x^2 + y^2) - 25x = 0$$
$$(3)\ \ 4(x^2 + y^2) - 25x = 0$$
$$(4)\ \ 4(x^2 + y^2) - 25x + 10 = 0$$

SOLUTION

First, we find an equation for which the point (3,4) is a solution

$$(1) \quad \left(2(x^2 + y^2) - 3x\right)\Big|_{x=3\ y=4} = 2(3^2 + 4^2) - 3 * 3 = 2 * 25 - 9 \neq 0$$

$$(2) \quad \left(3(x^2 + y^2) - 25x\right)\Big|_{x=3\ y=4} = 3(3^2 + 4^2) - 25 * 3 = 3 * 25 - 25 * 3 \equiv 0$$

$$(3) \quad \left(4(x^2 + y^2) - 25x\right)\Big|_{x=3\ y=4} = 4(3^2 + 4^2) - 25 * 3 = 4 * 25 - 25 * 3 \neq 0$$

$$(4) \quad \left(4(x^2 + y^2) - 25x + 10\right)\Big|_{x=3\ y=4} = 4(3^2+4^2)-25*3+10 = 4*25-25*3+10 \neq 0$$

we present the equation (2) to canonical form

$$3(x^2 + y^2) - 25x = 0 \longrightarrow x^2 + y^2 - \frac{25}{3}x = 0$$

$$x^2 - 2x\frac{25}{3 * 2} + \left(\frac{25}{2 * 3}\right)^2 - \left(\frac{25}{2 * 3}\right)^2 + y^2 = 0$$

$$\left(x - \frac{25}{6}\right)^2 + y^2 = \left(\frac{25}{6}\right)^2 \longrightarrow$$

This equation of the circle with center $\left(\frac{25}{6}, 0\right)$ and radius $R = \frac{25}{6}$. This means that the circle tangent to the y-axis at the origin.

ANSWER

$$(2)\ \ 3(x^2 + y^2) - 25x = 0$$

QUESTION №83

If $x+\frac{1}{x}=3$ then value of $x^5+\frac{1}{x^5}$ is

(1) 100 (2) 123 (3) 243 (4) 172

SOLUTION

$$\left(x+\frac{1}{x}\right)^5 = C_5^0x^5 + C_5^1\frac{x^4}{x} + C_5^2\frac{x^3}{x^2} + C_5^3\frac{x^2}{x^3} + C_5^4\frac{x^1}{x^4} + C_5^5\frac{x^0}{x^5} =$$

$$= x^5 + 5x^3 + 10x + 10\frac{1}{x} + 5\frac{1}{x^3} + \frac{1}{x^5} = x^5 + \frac{1}{x^5} + 5\left(x^3+\frac{1}{x^3}\right) + 10\left(x+\frac{1}{x}\right)$$

$$x^5+\frac{1}{x^5} = \left(x+\frac{1}{x}\right)^5 - 5\left(x^3+\frac{1}{x^3}\right) - 10\left(x+\frac{1}{x}\right)$$

$$\left(x+\frac{1}{x}\right)^3 = C_3^0x^3 + C_3^1\frac{x^2}{x} + C_3^2\frac{x^1}{x^2} + C_3^3\frac{x^0}{x^3} = x^3 + 3x + 3\frac{1}{x} + \frac{1}{x^3}$$

$$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right) = 3^3 - 3*3 = 27-9 = 18 = 2*3^2$$

$$x^5+\frac{1}{x^5} = \left(x+\frac{1}{x}\right)^5 - 5\left(x^3+\frac{1}{x^3}\right) - 10\left(x+\frac{1}{x}\right) = 3^5 - 5*2*3^2 - 10*3 =$$

$$= 3*81 - 3*30 - 10*3 = 3*(81-30-10) = 3*41 = 123$$

ANSWER

(2) 123