Answer on Question #56476 – Math – Trigonometry

Solve the following for $0 \le \theta \le 720°$

a) $3\cos\theta - 2 = 0$

Solution

for $0 \le \theta \le 720°$

$3\cos\theta - 2 = 0$
$3\cos\theta = 2$

$\cos\theta = 2/3$
$\theta = \arccos(2/3) + 2\pi n$, n=0,1, where arccos () is the inverse of cosine function.

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b) $2sin^2(\theta) - sin(\theta) - 1 = 0$

Solution

for $0 \le \theta \le 720°$

$2sin^2(\theta) - \sin(\theta) - 1 = 0$
Substitution $\sin(\theta)$=x

$2x^2 - x - 1 = 0$

D=1+4*2=9

$x_1 = \frac{1+3}{4} = 1,\ \sin(\theta) = 1,\ \theta = \frac{\pi}{2} + 2\pi n$, n=0,1; $\theta = \frac{\pi}{2};\ \frac{5\pi}{2}$

$x_2 = \frac{1-3}{4} = -\frac{1}{2},\quad \sin(\theta) = -\frac{1}{2},\ \ \theta = (-1)^{k+1}\frac{\pi}{6} + n\pi$, n=0,1,2, 3

$$\theta = \frac{\pi}{6};\ \frac{5\pi}{6};\ \frac{7\pi}{6};\ \frac{11\pi}{6};\ \frac{13\pi}{6};\ \frac{17\pi}{6};\ \frac{19\pi}{6};\ \frac{23\pi}{6}$$

Solve the following for $-2\pi \le \theta \le 2\pi$

a) $3\tan(2\theta) - 1 = 0$

Solution

for $-2\pi \le \theta \le 2\pi$

$3\tan(2\theta) - 1 = 0$
$3\tan(2\theta) = 1$

$\tan(2\theta) = 1/3$

$2\theta = \arctan(1/3) + \pi n$, n=-1,0,1

θ= 1/2· arctan(1/3)+πn/2, n=-2,-1,0,1, 2

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b) 12cos(2θ) – 6 = sinθ

Solution

for -2π ≤ θ ≤ 2π

12cos(2θ)– 6 = sinθ

use $sin^2\theta = \frac{1-\cos(2\theta)}{2} \implies \cos(2\theta) = 1 - 2sin^2(\theta)$

$12\ (1 - 2sin^2(\theta)) - 6 = \sin(\theta)$

$12 - 24sin^2(\theta) - 6 = \sin(\theta)$

$-24sin^2(\theta) - \sin(\theta) + 6 = 0$

$24sin^2(\theta) + \sin(\theta) - 6 = 0$

Substitution

$\sin(\theta) = x$

$24x^2 + x - 6 = 0$

$D = 1 + 4 \cdot 24 \cdot 6 = 577$

$x_1 = \frac{-1+\sqrt{577}}{2 \cdot 24} = \frac{-1+\sqrt{577}}{48}$

$\sin(\theta) = \frac{-1+\sqrt{577}}{48}$,

$\theta = (-1)^n \arcsin\left(\frac{-1+\sqrt{577}}{48}\right) + n\pi$, *n*=-1,0,1

$\theta = -\arcsin\left(\frac{-1+\sqrt{577}}{48}\right) - \pi;\ -\arcsin\left(\frac{-1+\sqrt{577}}{48}\right);\ \arcsin\left(\frac{-1+\sqrt{577}}{48}\right); \arcsin\left(\frac{-1+\sqrt{577}}{48}\right) + \pi\ ,$

$x_2 = \frac{-1+\sqrt{577}}{2 \cdot 24} = \frac{-1-\sqrt{577}}{48}$,

$\sin(\theta) = \frac{-1-\sqrt{577}}{48}$,

$\theta = (-1)^n arcsin\left(\frac{-1-\sqrt{577}}{48}\right) + n\pi$, *n*=-1,0,1

$\theta = -\arcsin\left(\frac{-1-\sqrt{577}}{48}\right) - \pi;\ -\arcsin\left(\frac{-1-\sqrt{577}}{48}\right);\ \arcsin\left(\frac{-1-\sqrt{577}}{48}\right); \arcsin\left(\frac{-1-\sqrt{577}}{48}\right) + \pi\ .$