

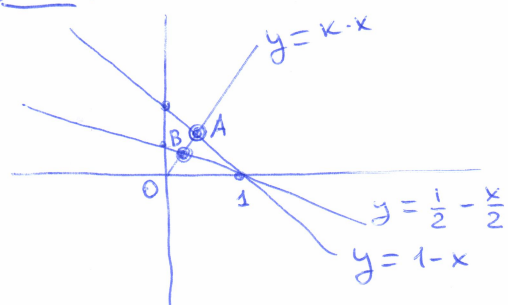
97.

$$2x^3 - 4x^2 + 7x - 5 = 2(x-1)\left(x^2 - x + \frac{5}{2}\right)$$

$a=1$; $b \cdot c = \frac{5}{2}$, $b+c=1$ by Viète theorem

$$\begin{aligned} & \frac{1}{(b-4)(a-2)+2a-4} + \frac{1}{(b-4)(c-2)+2c-4} + \frac{1}{(a-4)(c-2)+2c-4} = \\ & = \frac{1}{(b-2)(a-2)} + \frac{1}{(b-2)(c-2)} + \frac{1}{(a-2)(c-2)} = \\ & = \frac{c-2+b-2+a-2}{(a-2)(b-2)(c-2)} = \frac{a+b+c-6}{(a-2)(bc-2(b+c)+4)} = \\ & = \frac{1+1-6}{(-1)\left(\frac{5}{2}-2+4\right)} = \frac{8}{9} \end{aligned}$$

98.



$$x_A = \frac{1}{k+1}, \quad y_A = 1 - x_A = \frac{k}{k+1}$$

$$x_B = \frac{1}{2k+1}, \quad y_B = \frac{1}{2} - \frac{x_B}{2} = \frac{k}{2k+1}$$

$$OA = \sqrt{x_A^2 + y_A^2} = \frac{\sqrt{1+k^2}}{k+1}$$

$$OB = \sqrt{x_B^2 + y_B^2} = \frac{\sqrt{1+k^2}}{2k+1}$$

$$OA \cdot OB = \frac{1+k^2}{(k+1)(2k+1)}$$

$$OA \cdot OB = \frac{1}{3} \Rightarrow 3(1+k^2) = (k+1)(2k+1)$$

$$k^2 - 3k + 2 = 0$$

$$k_1 = 1, \quad k_2 = 2$$

$$k_1 + k_2 = 3$$

$$\boxed{99} \quad \sum_{k=1}^{\infty} \left(\frac{1-k}{2^k} \right) = \sum_{k=1}^{\infty} \frac{1}{2^k} - \sum_{k=1}^{\infty} \frac{k}{2^k}$$

$$S_1 := \sum_{k=1}^{\infty} \frac{1}{2^k} = \frac{1/2}{1-1/2} = 1$$

$$S_2 := \sum_{k=1}^{\infty} \frac{k}{2^k} = \frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \dots$$

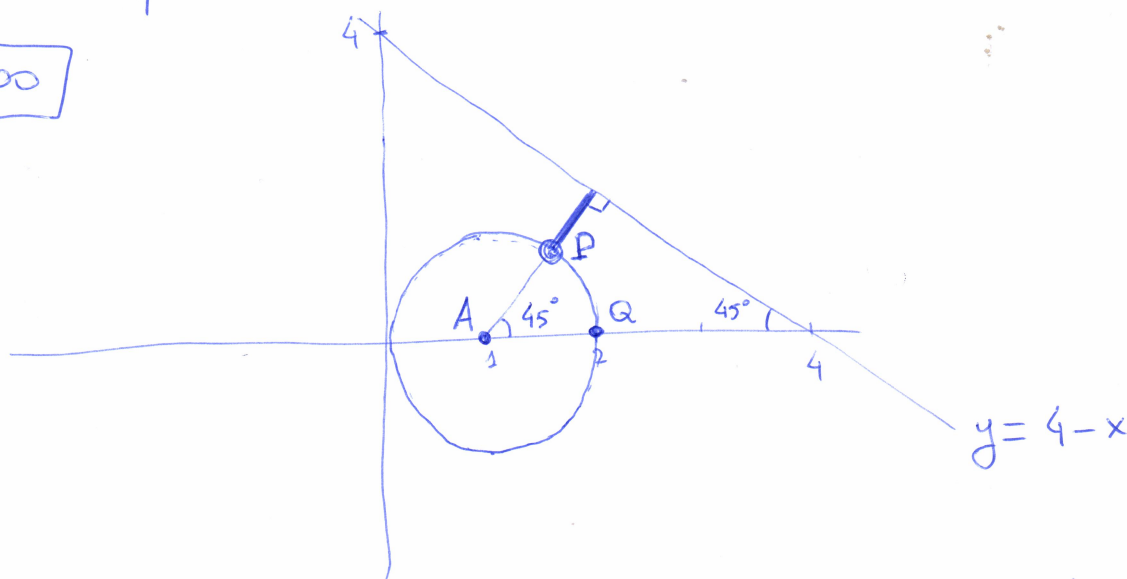
$$\frac{S_2}{2} = \frac{1}{2^2} + \frac{2}{2^3} + \frac{3}{2^4} + \dots$$

$$S_2 - \frac{S_2}{2} = \frac{1}{2^1} + (2-1)\frac{1}{2^2} + (3-2)\frac{1}{2^3} + \dots = \frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \dots =$$

$$= S_1 = 1 \Rightarrow S_2 = 2$$

$$S_1 - S_2 = -1$$

$\boxed{100}$



$$p^2 + q^2 - 2p = 0 \Leftrightarrow (p-1)^2 + q^2 = 1. \text{ this is a circle.}$$

$$P = \left(1 + \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right), \text{ because } \angle PAQ = 45^\circ \text{ (because AP is orthogonal to the line } y = 4 - x \text{)}$$

Distance from P to the line is:

$$\frac{\left| 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 4 \right|}{\sqrt{2}} = \frac{-\sqrt{2} + 3}{\sqrt{2}} = \frac{3}{\sqrt{2}} - 1$$