

## Answer on Question #56458 – Math – Statistics and Probability

### Question

The masses of a sample male frogs taken from a pond can be modeled by a normal distribution with a mean mass of 70g and standard deviation 5g. Four male frogs are chosen at random. Find the probability that their mean mass is less than 65g.

### Solution

Let  $\xi_k$ , where  $k = 1, 2, 3, 4$ , be mass of a male frog. It is given that  $\xi_k \sim N(70; 5)$ . Assume that they are independent identically distributed random variables.

Then we obtain

$$\xi_1 + \xi_2 + \xi_3 + \xi_4 \sim N(70 + 70 + 70 + 70; 5 + 5 + 5 + 5) = N(280; 20),$$

$$\text{hence } \frac{\xi_1 + \xi_2 + \xi_3 + \xi_4 - 280}{20} \sim N(0; 1).$$

So the required probability is equal to

$$\begin{aligned} P\left\{\frac{\xi_1 + \xi_2 + \xi_3 + \xi_4}{4} < 65\right\} &= P\{\xi_1 + \xi_2 + \xi_3 + \xi_4 < 260\} = P\{\eta < 260\} = P\left\{\frac{\eta - 280}{20} < \frac{260 - 280}{20}\right\} = \\ &= P\left\{\frac{\eta - 280}{20} < -1\right\} = 0.5 - \Phi(1) = 0.5 - 0.34134 = 0.15866. \end{aligned}$$

Here  $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{u^2}{2}} du$  is a tabulated function of Laplace and the value  $\Phi(1)$  was found from the table of Laplace.

**Answer:** 0.15866.