Answer on Question #56457 – Math – Algebra

92. *If* $\frac{1\underbrace{00\ldots01}_{n\ zeros}}{\underbrace{100\ldots01}_{(n+1)\ zeros}} < \frac{1\underbrace{00\ldots01}_{m\ zeros}}{\underbrace{100\ldots01}_{(m+1)\ zeros}}$ *then which of the following is true?*

(1) m>n (2) m<n (3) n>m+2 (4) n<m+2

Solution

$$\frac{1\underbrace{00\ldots01}_{n\ zeros}}{\underbrace{100\ldots01}_{(n+1)\ zeros}} = \frac{10^{n+1}+1}{10^{n+2}+1} = \frac{1}{10}\cdot\frac{10^{n+2}+10}{10^{n+2}+1} = \frac{1}{10}\cdot\frac{10^{n+2}+1+9}{10^{n+2}+1} =$$

$$= \frac{1}{10}\cdot\frac{10^{n+2}+1+9}{10^{n+2}+1} = \frac{1}{10}\cdot\left(1+\frac{9}{10^{n+2}+1}\right)$$

$$\frac{1\underbrace{00\ldots01}_{m\ zeros}}{\underbrace{100\ldots01}_{(m+1)\ zeros}} = \frac{1}{10}\cdot\left(1+\frac{9}{10^{m+2}+1}\right)$$

$$\frac{1\underbrace{00\ldots01}_{n\ zeros}}{\underbrace{100\ldots01}_{(n+1)\ zeros}} < \frac{1\underbrace{00\ldots01}_{m\ zeros}}{\underbrace{100\ldots01}_{(m+1)\ zeros}} \to \frac{1}{10}\cdot\left(1+\frac{9}{10^{n+2}+1}\right) < \frac{1}{10}\cdot\left(1+\frac{9}{10^{m+2}+1}\right)$$

$$1+\frac{9}{10^{n+2}+1} < 1+\frac{9}{10^{m+2}+1}$$

$$\frac{9}{10^{n+2}+1} < \frac{9}{10^{m+2}+1}$$

$$\frac{1}{10^{n+2}+1} < \frac{1}{10^{m+2}+1}$$

$$10^{n+2}+1 > 10^{m+2}+1$$

$$10^{n+2} > 10^{m+2}$$

$$n+2 > m+2$$

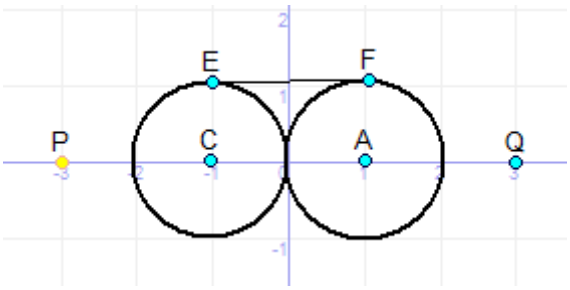
$$n > m$$

Answer: n>m.

93. *Consider a pair of circles* $(|x|-1)^2+|y|^2=1$. *If minimum length of path traced by a particle which starts from P(-3, 0) and reaches Q(3, 0) without entering inside any circle, is l, then*

(1) 6<l<7 (2) 7<l<8 (3) 8<l<9 (4) 9<l<10

Solution



EF = 2,

$1+\frac{2\pi r}{4} = 1+\frac{\pi r}{2} = 1+\frac{\pi}{2} = 2.57$ is a the length of a quarter of a circle.

The length of segment PE is

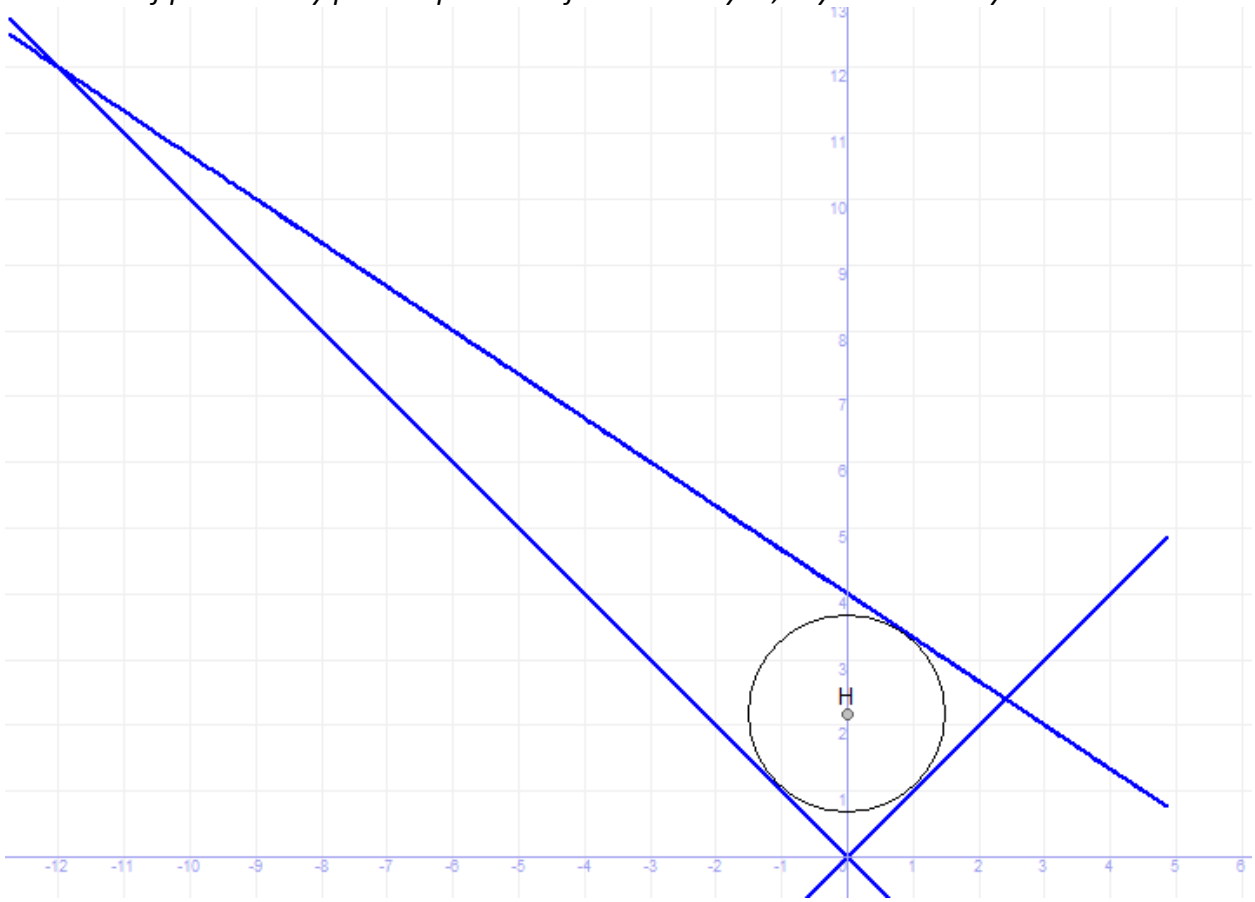
$$PE = \sqrt{PC^2+CE^2} = \sqrt{2^2+1^2} = 2.23$$

Then $2.57 > 2.23$.

The minimum length of path traced by a particle is $PE+EF+FQ=2.23+2+2.23=6.46$, $6< 6.46<7$.

Answer: $6<l<7$

94. *The number of points in x-y plane equidistant from lines x-y=0; x+y=0 and 2x+3y=12 are…*



Answer: 1.

95. *The graph of the function $y = 16x^2 + 8(a + 2)x - 3a - 2$ is strictly above the x-axis, then number of integral values of a is*

(1) 6 (2) 5 (3) 4 (4) 3

Solution

It's parabola. The graph of the parabola is strictly above the x-axis if and only if D<0

$$D = 64(a + 2)^2 + 64(3a + 2)$$
$$64(a + 2)^2 + 64(3a + 2) < 0$$
$$(a + 2)^2 + (3a + 2) < 0$$
$$a^2 + 4a + 4 + 3a + 2 < 0$$
$$a^2 + 7a + 6 < 0$$
$$a \in (-6; -1)$$

Integral values of a are -5, -4, -3, -2.

Answer: 4.

96. *Number of solution of the system of equations and inequations: a-b>-4, a-b<4 and a+b=16, a,b∈ I is…*

(1) 3 (2) 5 (3) 7 (4) Infinitely many

Solution

a+b=16 => a=16-b

a-*b*>-*4* => 16-2b>-4
a-*b*<*4* => 16-2b<4
-4<16-2b<4
-20<-2b<-12
6<b<10
b = 7, 8, 9
a = 9, 8, 7
Answer: 3.