# Answer on Question #56454 – Math – Algebra

**6.** How many solutions over the complex number system does this polynomial have?
2x^4 – 3x^3 – 24x^2 + 13x + 12 = 0
Enter your answer as an integer.

**Solution**

By Fundamental Theorem of Algebra, every polynomial of degree $n \geq 1$ has exactly $n$ zeros (counting multiplicities). In this problem $n = 4$. The solutions are x=-3, x=-1/2, x=1, x=4.
**Answer: 4.**

**7.** The value 4 is an upper bound for the zeros of the function shown below.
f(x) = 4x^3 – 12x^2 – x + 15
A: True
B: False

**Solution**

f(x) = 4x^3 – 12x^2 – x + 15.
There are two sign changes, hence, by Descart's rule of signs, there exist 2 or 0 positive zeros
f(-x) = -4x^3 – 12x^2 + x + 15.
There is only one sign change in this case, hence, by Descart's rule of signs, there is exactly one negative root.
The leading coefficient is 4, so we must divide all terms by 4
x^3 – 3x^2 – 0.25x + 3.75
The coefficients are 1; -3; -0.25; 3.75.
Drop the leading coefficient and remove any minus signs: 3; 0.25; 3.75
Bound 1: the largest value is 3.75. Plus 1 will be 4.75.
Bound 2: adding all values is 3+0.25+3.75=7. The sum of all values is greater than 1.
The smallest "bounds" value (4.75) is our answer. All roots are within plus or minus of that.
**Answer: B: False**

**8.** The value 0 is a lower bound for the zeros of the function shown below.
f(x) = -3x^3 + 20x^2 – 36x + 16
A: True
B: False

**Solution**

f(x) = -3x^3 + 20x^2 – 36x + 16
There are three sign changes, hence, by Descart's rule of signs, there exist 3 or 1 positive zeros.
f(-x) = 3x^3 + 20x^2 +36x + 16
There are zero sign changes, hence, by Descart's rule of signs, there are no negative zeros.
Therefore indeed 0 is a lower bound for the zeros of the function.

The leading coefficient is -3, so we must divide all terms by (-3)
x^3 – 20/3·x^2 +12x -16/3
The coefficients are 1; -20/3; 12; -16/3.
Drop the leading coefficient and remove any minus signs: 20/3; 12; 16/3

Bound 1: the largest value is 12. Plus 1 will be 13.
Bound 2: adding all values is 20/3+12+16/3=36/3+12=24.
The sum of all values is greater than 1. So the answer is 24.
The smallest "bounds" value (13) is our answer. All roots are within plus or minus of that. Observe that roots are $x = \frac{2}{3}$, $x = 2$, x=4.

**Answer: A: True**

**9.** Express the polynomial as a product of linear factors.
f(x) = 2x^3 + 4x^2 – 2x – 4
A: (x-4)(x+1)(x-1)
B: (x-2)(x-2)(x-1)
C: 2(x+2)(x+1)(x-1)
D: (x-2)(x+1)(x-1)

**Solution**

$f(x) = 2x^3 + 4x^2 - 2x - 4 = (2x^3 + 4x^2) - (2x + 4) = 2x^2(x+2) - 2(x+2) = 2(x+2)(x^2-1) = 2(x+2)(x-1)(x+1)$

**Answer: C: 2(x+2)(x+1)(x-1)**

**10.** What is the sum of the roots of the polynomial shown below?
f(x) = x^3 + 2x^2 – 11x – 12

**Solution**

**Method 1**

Using Vieta's formulas, the sum of th eroots of the polynomial is $x_1 + x_2 + x_3 = -\frac{2}{1} = -2$.

**Method 2**

Observe that the roots of the polynomial are x= –4, x=–1 and x=3 so the sum of the roots is equal to –4–1+3= -2.

**Answer:** –**2.**