Find the radius of a circle with center at C(1,1), if a chord of length 6 is bisected at M(3,4)
SOLUTION
In an x–y Cartesian coordinate system, the circle with centre coordinates (x_C, y_C) and radius R is the set of all points (x, y) such that

$$(x - x_C)^2 + (y - y_C)^2 = R^2$$

In our case

$$(x - 1)^2 + (y - 1)^2 = R^2$$

Let the points A and B (the beginning and end of the chord) have coordinates:

$$A(x_A, y_A),\ B(x_B, y_B)$$

Points A and B belong to the circle - which means that their coordinates satisfy the equation circle

$$\begin{cases} (x_A - 1)^2 + (y_A - 1)^2 = R^2 \\ (x_B - 1)^2 + (y_B - 1)^2 = R^2 \end{cases}$$

A chord AB of length 6:

$$(x_A - x_B)^2 + (y_A - y_B)^2 = 6^2$$

point M - the middle of segment AB. This means that its coordinates are expressed in terms of the coordinates of points A and B as follows:

$$\begin{cases} x_M = \dfrac{x_A + x_B}{2} \\ y_M = \dfrac{y_A + y_B}{2} \end{cases} \Rightarrow \begin{cases} 3 = \dfrac{x_A + x_B}{2} \\ 4 = \dfrac{y_A + y_B}{2} \end{cases} \Rightarrow \begin{cases} 6 = x_A + x_B \\ 8 = y_A + y_B \end{cases}$$

So, we have a system of five equations for the five unknowns $R,\ x_A,\ x_B,\ y_A,\ y_B$:

$$\begin{cases} (x_A - 1)^2 + (y_A - 1)^2 = R^2 \\ (x_B - 1)^2 + (y_B - 1)^2 = R^2 \\ (x_A - x_B)^2 + (y_A - y_B)^2 = 6^2 \\ 6 = x_A + x_B \\ 8 = y_A + y_B \end{cases}$$

$$\begin{cases} (x_A - 1)^2 + (y_A - 1)^2 = (x_B - 1)^2 + (y_B - 1)^2 \\ (x_A - x_B)^2 + (y_A - y_B)^2 = 6^2 \\ 6 = x_A + x_B \\ 8 = y_A + y_B \end{cases}$$

$$(x_A-1)^2+(y_A-1)^2=(x_B-1)^2+(y_B-1)^2 \rightleftarrows$$
$$(x_A-1)^2-(x_B-1)^2=(y_B-1)^2-(y_A-1)^2$$
$$((x_A-1)-(x_B-1))\left((x_A-1)+(x_B-1)\right)=$$
$$=((y_B-1)-(y_A-1))\left((y_B-1)+(y_A-1)\right)$$
$$(x_A-1-x_B+1)(x_A-1+x_B-1)=(y_B-1-y_A+1)(y_B-1+y_A-1) \rightleftarrows$$
$$(x_A-x_B)(x_A+x_B-2)=(y_B-y_A)(y_B+y_A-2)$$

$$\begin{cases} (x_A-x_B)(x_A+x_B-2)=(y_B-y_A)(y_B+y_A-2) \\ (x_A-x_B)^2+(y_A-y_B)^2=6^2 \\ 6=x_A+x_B \\ 8=y_A+y_B \end{cases}$$

$$\begin{cases} (x_A-x_B)(6-2)=(y_B-y_A)(8-2) \\ (x_A-x_B)^2+(y_A-y_B)^2=6^2 \\ 6=x_A+x_B \\ 8=y_A+y_B \end{cases}$$

$$\begin{cases} 4(x_A-x_B)=6(y_B-y_A) \\ (x_A-x_B)^2+(y_A-y_B)^2=6^2 \\ 6=x_A+x_B \\ 8=y_A+y_B \end{cases}$$

$$\frac{4}{9}(x_A-x_B)^2+(x_A-x_B)^2=6^2 \Rightarrow x_A-x_B=\sqrt{\frac{6^2*9}{9+4}}=\frac{3*6}{\sqrt{13}}$$

$$\begin{cases} \dfrac{3*6}{\sqrt{13}}=x_A-x_B \\ 6=x_A+x_B \end{cases} \Rightarrow x_A=3+\frac{9}{\sqrt{13}}$$

$$\frac{9}{4}(y_A-y_B)^2+(y_A-y_B)^2=6^2 \Rightarrow y_A-y_B=\sqrt{\frac{6^2*4}{9+4}}=\frac{2*6}{\sqrt{13}}$$

$$\begin{cases} \dfrac{2*6}{\sqrt{13}}=y_A-y_B \\ 8=y_A+y_B \end{cases} \Rightarrow y_A=4+\frac{6}{\sqrt{13}}$$

$$R=\sqrt{(x_A-1)^2+(y_A-1)^2}=\sqrt{\left(4+\frac{6}{\sqrt{13}}-1\right)^2+\left(3+\frac{9}{\sqrt{13}}-1\right)^2}=$$
$$=\sqrt{\left(3+\frac{6}{\sqrt{13}}\right)^2+\left(2+\frac{9}{\sqrt{13}}\right)^2}\approx 6.4783$$

ANSWER

$$R \approx 6.4783$$