

**Answer on Question #56355 – Math – Combinatorics | Number Theory**

There are 5 white, 4 yellow, 3 green, 2 blue and 1 red ball. The balls are identical except color. These are to be arranged in a line at 5 places. Find the no. Of distinct arrangement.

**Solution**

**Cases are as follows:**

- 1) 1 1 1 1 1, All balls are different colors**
- 2) 2 1 1 1, Two balls are the same color, three balls are different colors**
- 3) 2 2 1, Two balls are the same color, two balls are the same color, one ball is different colors**
- 4) 2 3, Two balls are the same color, three balls are the same color**
- 5) 3 1 1, Three balls are the same color, two balls are different colors**
- 6) 4 1, Four balls are the same color, one ball is different colors**
- 7) 5, All balls are the same color.**

**Corresponding numbers of arrangements:**

- 1) 5 different balls can be placed in 5 positions in**

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120 \text{ ways}$$

- 2) Pick the color of two balls from w, y, g, b:  $\binom{4}{1}$ ,**

**pick other 3 different balls from the rest colors:  $\binom{4}{3}$**

place 5 balls in 5 positions: 5!

Two balls are the same so we should divide by 2!

Thus

$$\frac{\binom{4}{3} \binom{4}{1} 5!}{2!} = 960,$$

And so on

$$3) \frac{\binom{4}{2} \binom{3}{1} 5!}{2! 2!} = 540,$$

$$4) \frac{\binom{3}{1} \binom{3}{1} 5!}{2! 3!} = 90,$$

$$5) \frac{\binom{3}{1} \binom{4}{2} 5!}{3!} = 360,$$

$$6) \frac{\binom{2}{1} \binom{4}{1} 5!}{4!} = 40,$$

$$7) 1.$$

$$\text{So } N = 120 + 960 + 540 + 90 + 360 + 40 + 1 = 2111.$$