

## Answer on Question #56316 – Math – Real Analysis

Test the convergence of the series:  $\frac{2}{1^3} - \frac{2}{2^3} + \frac{3}{3^3} + \frac{5}{4^3} + \dots$

### Solution

If  $a_1 = \frac{2}{1^3}$ ,  $a_2 = -\frac{2}{2^3}$ ,  $a_3 = \frac{3}{3^3}$ ,  $a_4 = \frac{5}{4^3}$  are terms of the given series, then we can rewrite  $a_n = \frac{x_n}{n^3}$ , where (for example)  $|x_n| \leq 3n$  for all  $n = 1, 2, \dots$ . Because the series

$$\sum_{n=1}^{\infty} \frac{3n}{n^3} = 3 \sum_{n=1}^{\infty} \frac{1}{n^2}$$

is convergent, the series

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{|x_n|}{n^3} \leq 3 \sum_{n=1}^{\infty} \frac{1}{n^2}$$

is also convergent by the Comparison Test. Thus, the series is absolutely convergent.

Hence, the series  $\sum_{n=1}^{\infty} a_n$  is convergent

**Answer:** the given series converges.