

## Answer on Question #56310 - Math – Analytic Geometry

Find the equation of the pair of straight lines joining the origin to the points of intersection of the circles represented by  $x^2 + y^2 = a^2$  and  $x^2 + y^2 + 2(gx + fy) = 0$ .

### Solution

Let's find points of intersection of these circles

$$\begin{cases} x^2 + y^2 = a^2 \\ x^2 + y^2 + 2(gx + fy) = 0 \end{cases}$$

$$\begin{cases} x^2 + y^2 = a^2 \\ a^2 + 2(gx + fy) = 0 \end{cases}$$

$$\begin{cases} x^2 + y^2 = a^2 \\ x = -\frac{a^2}{2g} - \frac{f}{g}y \end{cases}$$

$$\left(\frac{a^2}{2g} + \frac{f}{g}y\right)^2 + y^2 = a^2$$

$$\left(\frac{f^2}{g^2} + 1\right)y^2 + \frac{a^2f}{g^2}y + a^2\left(\frac{a^2}{4g^2} - 1\right) = 0$$

$$y = \frac{-\frac{a^2f}{g^2} \pm \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}}{2\left(\frac{f^2}{g^2} + 1\right)}$$

Thus

$$\begin{cases} x = \frac{-\frac{a^2}{g} \mp \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}}{2\left(\frac{f^2}{g^2} + 1\right)} \\ y = \frac{-\frac{a^2f}{g^2} \pm \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}}{2\left(\frac{f^2}{g^2} + 1\right)} \end{cases}$$

The equation of the line pathing through point

$$\left( \frac{-\frac{a^2}{g} - \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}}{2\left(\frac{f^2}{g^2} + 1\right)}, \frac{-\frac{a^2f}{g^2} + \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}}{2\left(\frac{f^2}{g^2} + 1\right)} \right)$$

is

$$y = \frac{\frac{a^2f}{g^2} - \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}}{\frac{a^2}{g} + \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}} x$$

And the of the line pathing through point

$$\left( \frac{-\frac{a^2}{g} + \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}}{2\left(\frac{f^2}{g^2} + 1\right)}, \frac{-\frac{a^2f}{g^2} - \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}}{2\left(\frac{f^2}{g^2} + 1\right)} \right)$$

is

$$y = \frac{\frac{a^2f}{g^2} + \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}}{\frac{a^2}{g} - \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}} x$$

**Answer:**  $y = \frac{\frac{a^2f}{g^2} - \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}}{\frac{a^2}{g} + \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}} x$

and

$$y = \frac{\frac{a^2f}{g^2} + \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}}{\frac{a^2}{g} - \sqrt{4a^2 + 4\frac{f^2a^2}{g^2} - \frac{a^4}{g^2}}} x$$