Answer on Question #56273 - Math - Complex Analysis

Find the number of solutions to the equation $z^2 + |z|^2 = 0$.

Solution

Put $z = x + iy$. Then the equation becomes

$$(x + iy)^2 + |x + iy|^2 = 0$$

$$x^2 + 2xyi - y^2 + x^2 + y^2 = 0,$$

so equating real and imaginary parts gives

$$\begin{cases} 2x^2 = 0, \\ 2xyi = 0. \end{cases}$$

If

$$2x^2 = 0 \rightarrow x^2 = 0 \rightarrow \boldsymbol{x = 0}.$$

If

$$2xyi = 0 \rightarrow \boldsymbol{x = 0 \ or \ y = 0}$$

Thus,

if $x = 0$, then $\boldsymbol{y = t}$**,** where t is an arbitrary real constant;

if $y = 0$, then it follows from the first equation of the system that $\boldsymbol{x} = 0$, but this case is a part of the previous condition, when $t = 0$.

So $\boldsymbol{z} = x + iy = 0 + ti = ti, t \in \mathbb{R}$, hence there are infinitely many solutions.

Answer: There are infinitely many solutions $z = ti, t \in \mathbb{R}$.