Answer on Question #56227 – Math – Other

Solve the following game

	x	y
x	2	5
y	4	1

Solution

	Player's B strategy		
Player's A strategy		x	y
	x	$a_{11} = 2$	$a_{12} = 5$
	y	$a_{21} = 4$	$a_{22} = 1$

<u>We associate x with 1, y with 2 in the next part of question.</u>

If A chooses the first row with probability p_1 (i.e. uses the mixed strategy $(p_1, p_2 = 1 - p_1)$), we equate his average return when B uses columns 1 and 2.

$$a_{11}p_1 + a_{21}(1 - p_1) = a_{12}p_1 + a_{22}(1 - p_1).$$

Solving for p_1, we find

$$p_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{21} + a_{12})}$$

Player A's average return using this strategy is

$$V = a_{11}p_1 + a_{21}(1 - p_1) = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11} + a_{22}) - (a_{21} + a_{12})}$$

If B chooses the first column with probability q_1 (i.e. uses the strategy $(q_1, q_2 = 1 - q_1)$), we equate his average losses when A uses rows 1 and 2.

$$a_{11}q_1 + a_{21}(1 - q_1) = a_{12}q_1 + a_{22}(1 - q_1).$$

Hence,

$$q_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{21} + a_{12})}$$

Player B's average loss using this strategy is

$$a_{11}q_1 + a_{21}(1 - q_1) = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11} + a_{22}) - (a_{21} + a_{12})}$$

The following formulae are used to find the value of the game and optimum strategies:

$$p_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{21} + a_{12})}; p_2 = 1 - p_1$$

$$q_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{21} + a_{12})}; q_2 = 1 - q_1$$

and the value of the game is

$$V = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11} + a_{22}) - (a_{21} + a_{12})}.$$

Therefore,

$$p_1 = \frac{1-4}{(2+1)-(4+5)} = \frac{1}{2}; p_2 = 1 - \frac{1}{2} = \frac{1}{2}.$$

$$q_1 = \frac{1-5}{(2+1)-(4+5)} = \frac{2}{3}; q_2 = 1 - \frac{2}{3} = \frac{1}{3}$$

and the value of the game is

$$V = \frac{2 \cdot 1 - 5 \cdot 4}{(2+1)-(4+5)} = 3.$$

The value of the game is 3, the optimal strategy for A is $\left(\frac{1}{2}, \frac{1}{2}\right)$ and the optimal strategy for B is $\left(\frac{2}{3}, \frac{1}{3}\right)$.