Answer on Question #56215 – Math – Real Analysis

Question

Prove by method of contradiction that α is an irrational number and ß is a rational number, then α+ß is an irrational number.

Proof

Let α be irrational and β be rational. To strive for a contradiction, assume that $\alpha + \beta$ is rational. By definition of rational numbers, $\alpha + \beta = \frac{m}{n}$ for some $m \in \mathbb{Z}, n \in \mathbb{N}$. Also, since β is rational, $\beta = \frac{m_1}{n_1}$, for appropriate numbers $m_1 \in \mathbb{Z}, n_1 \in \mathbb{N}$. But then $\alpha = \frac{m}{n} - \frac{m_1}{n_1} = \frac{m \cdot n_1 - n \cdot m_1}{n \cdot n_1}$ is a rational number, which contradicts with the assumption that α is irrational.