Answer on Question #56185 – Math – Algebra

1. State the x-coordinate of the x-intercept in quadrant 2 for the function below.

$f(x) = 4x^3 - 12x^2 - x + 15$

Solution

We guess a root $x = -1$.

Divide

$f(x) = 4x^3 - 12x^2 - x + 15$

by $(x + 1)$ and obtain $4x^2 - 16x + 15$.

Solving

$$4x^2 - 16x + 15 = 0$$

$D = (-16)^2 - 4 \cdot 4 \cdot 15 = 16^2 - 16 \cdot 15 = 16 \cdot (16 - 15) = 16$

$x_1 = \frac{16-4}{2\cdot4} = \frac{12}{8} = \frac{3}{2} = 1.5,$

$x_2 = \frac{16+4}{2\cdot4} = \frac{20}{8} = \frac{5}{2} = 2.5.$

Roots of

$f(x) = 4x^3 - 12x^2 - x + 15$

are as follows:

x=-1;

x=1.5;

x=2.5.

Only x=-1 is in quadrant 2.

Answer: -1.

2. State the y-coordinate of the y-intercept for the function below.

$f(x) = 4x^3 - 12x^2 - x + 15$

Solution

y-intercept= f(0) = $4 \cdot 0^3 - 12 \cdot 0^2 - 0 + 15$ =15

Answer: 15.

3. What is the maximum number of turns in the graph of this function?

f(x) = $4x^3 - 12x^2 - x + 15$

Solution

It is a third degree polynomial, then the maximum number of turns is two.

Answer: 2.

4. What is the local maximum value of the function? (Round answer to the nearest thousandth) g(x) =$x^3+5x^2-17x-21$

Solution

g'(x) =$3x^2+10x-17$

If $3x^2+10x-17=0$, then $D=10^2-4\cdot3\cdot(-17)=304$

$$x_1=\frac{-10+\sqrt{304}}{2\cdot3}=\frac{-5+\sqrt{76}}{3}$$

$$x_1=\frac{-10+\sqrt{304}}{2\cdot3}=\frac{-5-\sqrt{76}}{3}$$

g'=0 if $x=-\frac{5}{3}\pm\frac{2\sqrt{19}}{3}$,

g'>0 if $x<-\frac{5}{3}-\frac{2\sqrt{19}}{3}$,

g'<0 if $-\frac{5}{3}-\frac{2\sqrt{19}}{3}<x<-\frac{5}{3}+\frac{2\sqrt{19}}{3}$

g'>0 if $x>-\frac{5}{3}+\frac{2\sqrt{19}}{3}$.

So, $x_{max}=-\frac{5}{3}-\frac{2\sqrt{19}}{3}$ corresponds to local maximum.

Then, $g_{max}=g(x_{max})=x_{max}^3+5x_{max}^2-17x_{max}-21=\left(-\frac{5}{3}-\frac{2\sqrt{19}}{3}\right)^3+5\left(-\frac{5}{3}-\frac{2\sqrt{19}}{3}\right)^2-17\left(-\frac{5}{3}-\frac{2\sqrt{19}}{3}\right)-21=\frac{16}{27}(28+19\sqrt{19})\cong65.671$

Answer: 65.671.