

Answer on Question #56103 – Math - Algebra

Question 1. Which of the following is a root of the polynomial shown below?

$$f(x) = x^3 + 2x^2 - x - 2$$

- A: 2
- B: 0
- C: 3
- D: 1

Solution

The correct answer is "D: 1", because $f(1) = 1^3 + 2 \cdot 1^2 - 1 - 2 = 0$.

Question 2. Which of the following represents the set of possible rational roots for the polynomial shown below?

$$2x^3 + 5x^2 - 8x - 10 = 0$$

- A: $\{\pm \frac{1}{2}, \pm 1, \pm 2, \pm \frac{5}{2}, \pm 5, \pm 10\}$
- B: $\{\frac{1}{2}, 1, 2, \frac{5}{2}, 4, 5, 10, 20\}$
- C: $\{\pm \frac{2}{5}, \pm \frac{1}{2}, \pm 1, \pm 2, \pm \frac{2}{5}, \pm \frac{1}{5}, \pm \frac{1}{10}\}$
- D: $\{\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20\}$

Solution

Answer: A: $\{\pm \frac{1}{2}, \pm 1, \pm 2, \pm \frac{5}{2}, \pm 5, \pm 10\}$, because $x_1 x_2 x_3 = 5$ and $x_1 + x_2 + x_3 = -\frac{5}{2}$ according to Viet's formula, where x_1, x_2, x_3 are the roots of the equation $2x^3 + 5x^2 - 8x - 10 = 0$.

Question 3. Which of the following expresses the possible number of positive real solutions for the polynomial equation shown below?

$$X^3 - 4x^2 - 7x + 28 = 0$$

- A: one
- B: Three or one
- C: Two or Zero
- D: Two

Solution

Answer: B: Three or one. If a polynomial with real coefficients has a root which is a complex number, then the conjugate complex number is also a root. Thus, polynomials of degree 3 may possess either 3 real roots, or 1 real and 2 complex roots.

Question 4. A polynomial has one root that equals $5 - 7i$. Name one other root of this polynomial.

Solution

Answer: $5 + 7i$ (assuming coefficients of a polynomial to be real).

If a polynomial with real coefficients possesses a root which is a complex number, then the conjugate complex number is also a root.

Question 5. If a polynomial has one root in the form $a + \sqrt{b}$, it has a second root in the form of a _____ \sqrt{b}

Solution

Answer: $a - \sqrt{b}$ (assuming b to be negative). If b is negative, then $a + \sqrt{b}$ is a complex number.