Answer on Question #56023 – Math – Algebra

During a research experiment, it was found that the number of bacteria in a culture grew at a rate proportional to its size. At 10:00 AM there were 5,00 bacteria present in the culture. At noon, the number of bacteria grew to 5,800. How many bacteria will there be at midnight?

Solution

Denote the number of bacteria in a culture (which depends on time t) as $x(t)$. Then the rate of bacteria growth is $\frac{dx}{dt}$. We have

$$\frac{dx}{dt} = kx,$$

where k is the corresponding constant of proportionality. Then

$$\frac{dx}{x} = kdt,$$
$$ln|x| = kt,$$
$$x = C \cdot e^{kt}$$

Letting that $t_0 = 0$, $t_1 = 12 - 10 = 2$, the constants C and k can be found from the next system:

$$x(0) = C = 5{,}00; \;\; x(2) = C \cdot e^{2k} = 5{,}800,$$

Dividing 5,800 by 5,00 we get

$$e^{2k} = 11.6$$

At midnight the number of bacteria will be

$x(t_2) = x(24 - 10) = x(14) = 5{,}00 \cdot e^{14k} = = 5{,}00 \cdot (e^{2k})^7 = 5{,}00 \cdot (11.6)^7 \approx$
$\approx 14{,}131{,}098{,}670.$

Answer: $14{,}131{,}098{,}670$ bacteria.