Answer on Question #56010 – Math – Linear Algebra

Let $A = 2i - j + k$, $B = i + 3j - 2k$, $C = -2i + j - 2k$, and $D = 3i + 2j + 5k$.
Find scalar a, b, c such that $D = aA + bB + cC$

Solution

First find aA, bB, cC in component form

$$aA = a(2i - j + k) = 2ai - aj + ak$$

$$bB = b(i + 3j - 2k) = bi + 3bj - 2bk$$

$$cC = c(-2i + j - 2k) = -2ci + cj - 2ck$$

Now let's sum these equations and compare them with the expression for D in component form

$$D = aA + bB + cC = 2ai - aj + ak + bi + 3bj - 2bk - 2ci + cj - 2ck$$
$$= (2a + b - 2c)i + (-a + 3b + c)j + (a - 2b - 2c)k$$

$$D = 3i + 2j + 5k$$

Equating expressions that stood near the corresponding unit vectors, we obtain the following system

$$\begin{matrix} i: \\ j: \\ k: \end{matrix} \begin{cases} 2a + b - 2c = 3 \\ -a + 3b + c = 2 \\ a - 2b - 2c = 5 \end{cases}$$

Express *b* from the first equation and substitute for *b* into the second and the third equations of the system:

$$\begin{cases} b = 3 + 2c - 2a \\ -a + 9 + 6c - 6a + c = 2 \\ a - 6 - 4c + 4a - 2c = 5 \end{cases}$$

$$\begin{cases} b = 3 + 2c - 2a \\ 7c - 7a = -7 \\ 5a - 6c = 11 \end{cases}$$

Divide the second equation by 7:

$$\begin{cases} b = 3 + 2c - 2a \\ c - a = -1 \\ 5a - 6c = 11 \end{cases}$$

Express *c* from the second equation and substitute for *c* into the third equations of the system:

$$\begin{cases} b = 3 + 2c - 2a \\ c = -1 + a \\ 5a - 6a + 6 = 11 \end{cases}$$

$$\begin{cases} b = 3 + 2c - 2a \\ c = -1 + a \\ -a = 5 \end{cases}$$

$$\begin{cases} b = 3 + 2c - 2a \\ c = -6 \\ a = -5 \end{cases}$$

$$\begin{cases} b = 1 \\ c = -6 \\ a = -5 \end{cases}$$

Answer: $a = -5,\ b = 1,\ c = -6.$