# Answer on Question #55954 - Math – Complex Analysis

If $z = \mathbf{cos}\,\theta + j\,\mathbf{sin}\,\theta$, prove that when $n$ is a natural number

$\mathbf{sin}\,n\theta = \frac{1}{2j}\left(z^n - \frac{1}{z^n}\right)$.

## Solution

$$\boldsymbol{z} = \cos\boldsymbol{\theta} + \boldsymbol{j}\sin\boldsymbol{\theta} = e^{j\theta}$$

According to Euler's formula:

$e^{jn\theta} = \cos(\boldsymbol{n\theta}) + \boldsymbol{j}\sin(\boldsymbol{n\theta})$

$e^{-jn\theta} = \cos(-\boldsymbol{n\theta}) + \boldsymbol{j}\sin(-\boldsymbol{n\theta}) = \cos(\boldsymbol{n\theta}) - \boldsymbol{j}\sin(\boldsymbol{n\theta})$.

Subtracting the second formula from the first one and dividing by $(2j)$ obtain

$$\sin(\boldsymbol{n\theta}) = \frac{e^{jn\theta} - e^{-jn\theta}}{2j}.$$

On the other hand,

$$e^{jn\theta} - e^{-jn\theta} = \left(e^{j\theta}\right)^n - \left(e^{j\theta}\right)^{-n} = z^n - z^{-n} = z^n - \frac{1}{z^n}$$

Thus,

$$\sin\boldsymbol{n\theta} = \frac{\mathbf{1}}{\mathbf{2}\boldsymbol{j}}\left(\boldsymbol{z^n} - \frac{\mathbf{1}}{\boldsymbol{z^n}}\right).$$