A bag contains three red and three green balls and a person draws out three at random. He then drops three blue balls into the bag and again draws out three at random. The chance that 3 later balls being all of different colors is

Solution

Let $E =$ "3 later balls being all of different colors",

A="3 red balls were drawn out at random",

B="2 red and 1 green balls were drawn out at random",

C="2 red and 1 green balls were drawn out at random",

D="3 green balls were drawn out at random".

After the insertion of three blue balls the bag may contain:

	Red	Green	Blue
A	3	0	3
B	2	1	3
C	1	2	3
D	0	3	3

The probability that the bag contains balls according to case A from the above table is

$$P(A) = \frac{3!}{0!\,(3-0)!}0.5^0 0.5^{3-0} = \frac{1}{8}.$$

The probability that the bag contains balls according to case B from the above table is

$$P(B) = \frac{3!}{1!\,(3-1)!}0.5^1 0.5^{3-1} = \frac{3}{8}.$$

The probability that the bag contains balls according to case C from the above table is

$$P(C) = \frac{3!}{2!\,(3-2)!}0.5^2 0.5^{3-2} = \frac{3}{8}.$$

The probability that the bag contains balls according to case D from the above table is

$$P(D) = \frac{3!}{3!\,(3-3)!}0.5^3 0.5^{3-3} = \frac{1}{8}.$$

Now three different colors on the second draw can only be obtained if the six balls come under cases (B) and (C).

Under (B) at the second draw the probability of drawing three different colored balls is

$$P_2 = P(GRB)N(possible\ ways\ of\ permutations\ of\ GRB) = \frac{1}{6}\cdot\frac{2}{5}\cdot\frac{3}{4}\cdot 6 = \frac{3}{10}.$$

Therefore the probability that this will happen is

$$P(B) \cdot P_2 = \frac{3}{8} \cdot \frac{3}{10} = \frac{9}{80}.$$

Under (C) the probability $P(C) \cdot P_3$ is the same.

The total chance is the sum of these probabilities and amounts to

$$P(E) = \frac{9}{80} + \frac{9}{80} = \frac{9}{40}.$$