Answer on Question #55839 – Math – Linear Algebra

6. Solve the set of linear equations by Guassian elimination method : a+2b+3c=5, 3a-b+2c=8, 4a-6b-4c=-2. Find c

4

5

9

10

Solution

$$\begin{cases} a+2b+3c=5 \\ 3a-b+2c=8 \\ 4a-6b-4c=-2 \end{cases}$$

In matrix form:

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 3 & -1 & 2 & 8 \\ 4 & -6 & -4 & -2 \end{array}\right)$$

1st row is multiplied by 3 and subtracted from 2nd row; 1st row is multiplied by 4 and subtracted from 3rd row

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -7 & -7 & -7 \\ 0 & -14 & -16 & -22 \end{array}\right)$$

2nd row is divided by 7

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & -14 & -16 & -22 \end{array}\right)$$

2nd row is multiplied by 2 and subtracted from 1st row; 2nd row is multiplied by 14 and added to 3rd row

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -8 \end{array}\right)$$

3rd row is divided by -2

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

3rd row is subtracted from 2nd row

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

3rd row is subtracted from 1st row

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

$$\begin{cases} a = -1 \\ b = -3 \\ c = 4 \end{cases}$$

Answer: c = 4

7. Solve the set of linear equations by the matrix method : a+3b+2c=3 , 2a-b-3c= -8, 5a+2b+c=9. Sove for b

9

-3

5

-4

Solution

$$\begin{cases} a + 3b + 2c = 3 \\ 2a - b - 3c = -8 \\ 5a + 2b + 1c = 9 \end{cases}$$

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{pmatrix};\ B = \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix};\ X = \begin{pmatrix} a \\ b \\ c \end{pmatrix};$$

$A \cdot X = B;$

$X = A^{-1} \cdot B$

Let's find the inverse of A.

$$\det(A) = \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} -1 & -3 \\ 2 & 1 \end{vmatrix} - 3 \cdot \begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} + 2 \cdot \begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} =$$

$$= (-1) \cdot 1 - 2 \cdot (-3) - 3[2 \cdot 1 - 5 \cdot (-3)] + 2 \cdot [2 \cdot 2 - 5 \cdot (-1)] = -1 + 6 - 3[2 + 15] + 2 \cdot [4 + 5] = 23 - 51 = -28$$

det (*A*) = -28

$$M_{11} = \begin{vmatrix} -1 & -3 \\ 2 & 1 \end{vmatrix} = -1 \cdot 1 - 2 \cdot (-3) = 5;$$

$$A_{11} = (-1)^{1+1} M_{11} = 5;$$

$$M_{12} = \begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} = 2 \cdot 1 - 5 \cdot (-3) = 17;$$

$$A_{12} = (-1)^{1+2} M_{12} = -17;$$

$$M_{13} = \begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} = 2 \cdot 2 - 5 \cdot (-1) = 9;$$

$$A_{13} = (-1)^{1+3} M_{13} = 9;$$

$$M_{21} = \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = 3 \cdot 1 - 2 \cdot 2 = -1;$$

$$A_{21} = (-1)^{2+1} M_{21} = 1;$$

$$M_{22} = \begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} = 1 \cdot 1 - 5 \cdot 2 = -9;$$

$$A_{22} = (-1)^{2+2} M_{22} = -9;$$

$$M_{23} = \begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix} = 1 \cdot 2 - 5 \cdot 3 = -13;$$

$$A_{23} = (-1)^{2+3} M_{23} = 13;$$

$$M_{31} = \begin{vmatrix} 3 & 2 \\ -1 & -3 \end{vmatrix} = 3 \cdot (-3) - (-1) \cdot 2 = -7;$$

$$A_{31} = (-1)^{3+1} M_{31} = 7;$$

$$M_{32} = \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = 1 \cdot (-3) - 2 \cdot 2 = -7;$$

$$A_{32} = (-1)^{3+2} M_{32} = 7;$$

$$M_{33} = \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = 1 \cdot (-1) - 2 \cdot 3 = -7;$$

$$A_{33} = (-1)^{3+3} M_{33} = -7;$$

Cofactor matrix:

$$C^* = \begin{pmatrix} 5 & -17 & 9 \\ 1 & -9 & 13 \\ 7 & 7 & -7 \end{pmatrix}$$

Adjugate:

$$C^{*T} = \begin{pmatrix} 5 & 1 & 7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}$$

Inverse of A:

$$A^{-1} = \frac{C^{*T}}{\det A} = \begin{pmatrix} -\frac{5}{28} & -\frac{1}{28} & \frac{1}{4} \\ \frac{17}{28} & \frac{9}{28} & -\frac{1}{4} \\ -\frac{9}{28} & -\frac{13}{28} & \frac{1}{4} \end{pmatrix}$$

The solution:

$$X = A^{-1} \cdot B = \begin{pmatrix} -\frac{5}{28} & -\frac{1}{28} & \frac{1}{4} \\ \frac{17}{28} & \frac{9}{28} & -\frac{1}{4} \\ -\frac{9}{28} & -\frac{13}{28} & \frac{1}{4} \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix};$$

$$\begin{cases} a = 2 \\ b = -3 \\ c = 5 \end{cases}$$

Answer: b = -3

8. Solve the set of linear equations by Guassian elimination method : a+2b+3c=5, 3a-b+2c=8, 4a-6b-4c=-2. Find b

4

-5

-3

5

Solution

$$\begin{cases} a+2b+3c=5 \\ 3a-b+2c=8 \\ 4a-6b-4c=-2 \end{cases}$$

In matrix form:

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 3 & -1 & 2 & 8 \\ 4 & -6 & -4 & -2 \end{array}\right)$$

1st row is multiplied by 3 and subtracted from 2nd row; 1st row is multiplied by 4 and subtracted from 3rd row

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -7 & -7 & -7 \\ 0 & -14 & -16 & -22 \end{array}\right)$$

2nd row is divided by -7

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & -14 & -16 & -22 \end{array}\right)$$

2nd row is multiplied by 2 and subtracted from 1st row; 2nd row is multiplied by 14 and added to 3rd row

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -8 \end{array}\right)$$

3rd row is divided by -2

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

3rd row is subtracted from 2nd row

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

3rd row is subtracted from 1st row

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

$$\begin{cases} a = -1 \\ b = -3 \\ c = 4 \end{cases}$$

Answer: b = -3

9. Solve the set of linear equations by Guassian elimination method : x+2y+3z=5, 3x-y+2z=8, 4x-6y-4=-2. Find a

-1

4

5

-11

Solution

$$\begin{cases} a + 2b + 3c = 5 \\ 3a - b + 2c = 8 \\ 4a - 6b - 4c = -2 \end{cases}$$

In matrix form:

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 3 & -1 & 2 & 8 \\ 4 & -6 & -4 & -2 \end{array}\right)$$

1st row is multiplied by 3 and subtracted from 2nd row; 1st row is multiplied by 4 and subtracted from 3rd row

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -7 & -7 & -7 \\ 0 & -14 & -16 & -22 \end{array}\right)$$

2nd row is divided by -7

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 1 \\ 0 & -14 & -16 & -22 \end{array}\right)$$

2nd row is multiplied by 2 and subtracted from 1st row; 2nd row is multiplied by 14 and added to 3rd row

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -8 \end{array}\right)$$

3rd row is divided by -2

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

3rd row is subtracted from 2nd row

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

3rd row is subtracted from 1st row

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

$$\begin{cases} a = -1 \\ b = -3 \\ c = 4 \end{cases}$$

Answer: a = -1.

10. Solve the set of linear equations by the matrix method : a+3b+2c=3 , 2a-b-3c= -8, 5a+2b+c=9. Sove for a

2

4

7

3

Solution

$$\begin{cases} a+3b+2c=3 \\ 2a-b-3c=-8 \\ 5a+2b+1c=9 \end{cases}$$

$$A=\begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{pmatrix};\ B=\begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix};\ X=\begin{pmatrix} a \\ b \\ c \end{pmatrix};$$

$$A\cdot X=B;$$

$$X=A^{-1}\cdot B$$

Let’s find the inverse of A.

$$\det(A)=\begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{vmatrix}=1\cdot\begin{vmatrix} -1 & -3 \\ 2 & 1 \end{vmatrix}-3\cdot\begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix}+2\cdot\begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix}=$$

$$=(-1)\cdot 1-2\cdot(-3)-3[2\cdot 1-5\cdot(-3)]+2\cdot[2\cdot 2-5\cdot(-1)]=-1+6-3[2+15]+2\cdot[4+5]=23-51=-28$$

det (*A*) = -28

$$M_{11}=\begin{vmatrix} -1 & -3 \\ 2 & 1 \end{vmatrix}=-1\cdot 1-2\cdot(-3)=5;$$

$$A_{11}=(-1)^{1+1}M_{11}=5;$$

$$M_{12}=\begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix}=2\cdot 1-5\cdot(-3)=17;$$

$$A_{12}=(-1)^{1+2}M_{12}=-17;$$

$$M_{13} = \begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} = 2 \cdot 2 - 5 \cdot (-1) = 9;$$

$$A_{13} = (-1)^{1+3} M_{13} = 9;$$

$$M_{21} = \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = 3 \cdot 1 - 2 \cdot 2 = -1;$$

$$A_{21} = (-1)^{2+1} M_{21} = 1;$$

$$M_{22} = \begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} = 1 \cdot 1 - 5 \cdot 2 = -9;$$

$$A_{22} = (-1)^{2+2} M_{22} = -9;$$

$$M_{23} = \begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix} = 1 \cdot 2 - 5 \cdot 3 = -13;$$

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$$M_{31} = \begin{vmatrix} 3 & 2 \\ -1 & -3 \end{vmatrix} = 3 \cdot (-3) - (-1) \cdot 2 = -7;$$

$$A_{31} = (-1)^{3+1} M_{31} = 7;$$

$$M_{32} = \begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = 1 \cdot (-3) - 2 \cdot 2 = -7;$$

$$A_{32} = (-1)^{3+2} M_{32} = 7;$$

$$M_{33} = \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = 1 \cdot (-1) - 2 \cdot 3 = -7;$$

$$A_{33} = (-1)^{3+3} M_{33} = -7;$$

Cofactor matrix:

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The solution:

$$X = A^{-1} \cdot B = \begin{pmatrix} -\frac{5}{28} & -\frac{1}{28} & \frac{1}{4} \\ \frac{17}{28} & \frac{9}{28} & -\frac{1}{4} \\ -\frac{9}{28} & -\frac{13}{28} & \frac{1}{4} \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix};$$

$$\begin{cases} a = 2 \\ b = -3 \\ c = 5 \end{cases}$$

Answer: a = 2.