Answer on Question #55838 – Math – Linear Algebra

1. Solve for x and y: 3x + 4y = 9, 2x + 3y =8

x = 7.5, y = - 4. 5
x = 7.0, y = - 4. 5
x = 4.5, y = - 7. 5
x = 7.5, y = - 4. 1

Solution

$$\begin{cases} 3x + 4y = 9 \\ 2x + 3y = 8 \end{cases}$$

$$\begin{vmatrix} 3 & 4 & |9 \\ 2 & 3 & |8 \end{vmatrix}$$

Using Cramer's rule

$$\Delta = \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} = 3 \cdot 3 - 2 \cdot 4 = 9 - 8 = 1$$

$$\Delta_x = \begin{vmatrix} 9 & 4 \\ 8 & 3 \end{vmatrix} = 9 \cdot 3 - 8 \cdot 4 = 27 - 32 = -5$$

$$\Delta_y = \begin{vmatrix} 3 & 9 \\ 2 & 8 \end{vmatrix} = 3 \cdot 8 - 2 \cdot 9 = 24 - 18 = 6$$

$$x = \frac{\Delta_x}{\Delta} = \frac{-5}{1} = -5$$

$$y = \frac{\Delta_y}{\Delta} = \frac{6}{1} = 6$$

Answer: x=-5, y=6

2. If A.x=λx, where A= ||||211232−212||||, determine the eigen values of the matrix A, and an eigen vector corresponding to each eigen value.
If
λ=2
,what is b

Solution

$$\hat{A}x = \lambda x$$

$$\hat{A} = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ -2 & 1 & 2 \end{bmatrix}$$

Let’s determine the eigenvalues of matrix A:

$$\hat{A}x = \lambda\hat{E}x$$

$$\hat{A}x - \lambda\hat{E}x = 0$$

$$\left(\hat{A} - \lambda\hat{E}\right)x = 0$$

$$det\begin{bmatrix} 2-\lambda & 1 & 1 \\ 2 & 3-\lambda & 2 \\ -2 & 1 & 2-\lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 2 & 3-\lambda & 2 \\ -2 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\cdot(3-\lambda)\cdot(2-\lambda) + 1\cdot 2\cdot(-2) + 2\cdot 1\cdot 1 - (-2)\cdot(3-\lambda)\cdot 1 - 1\cdot 2\cdot(2-\lambda) - 1\cdot 2 \cdot (2-\lambda) = 0$$

$$(2-\lambda)^2\cdot(3-\lambda) - 4 + 2 + 6 - 2\lambda + 4\lambda - 8 = 0$$

$$(2-\lambda)^2\cdot(3-\lambda) + 2\lambda - 4 = 0$$

$$(2-\lambda)^2\cdot(3-\lambda) - 2(2-\lambda) = 0$$

$$(2-\lambda)\cdot\left((2-\lambda)\cdot(3-\lambda) - 2\right) = 0$$

$$(2-\lambda)\cdot(\lambda^2 - 5\lambda + 6 - 2) = 0$$

$$(2-\lambda)\cdot(\lambda^2 - 5\lambda + 4) = 0$$

$$\lambda_1 = 2$$

$$\lambda^2 - 5\lambda + 4 = 0$$

$$D = 25 - 16 = 9$$

$$\lambda_2 = 4 \;\; \lambda_3 = 1$$

$$\lambda = 1; \lambda = 2, \lambda = 4$$

Determine the eigen vector corresponding to each eigen value:

$\lambda = 1$:

$$\begin{vmatrix} 2-1 & 1 & 1 \\ 2 & 3-1 & 2 \\ -2 & 1 & 2-1 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -2 & 1 & 1 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 3 & 3 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 3 & 3 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ 0 + x_2 + x_3 = 0 \end{cases} ==> \begin{cases} x_1 = 0 \\ x_2 + x_3 = 0 \end{cases}$$

$$x_1 = 0$$
$$x_2 = -x_3$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, x_3 \neq 0.$$

$\lambda = 2$:

$$\left| \begin{matrix} 2-2 & 1 & 1 \\ 2 & 3-2 & 2 \\ -2 & 1 & 2-2 \end{matrix} \right| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \Bigg| = 0$$

$$\left| \begin{matrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ -2 & 1 & 0 \end{matrix} \right| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \Bigg| = \left| \begin{matrix} 0 & 1 & 1 \\ 1 & \frac{1}{2} & 1 \\ 0 & 2 & 2 \end{matrix} \right| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \Bigg| = \left| \begin{matrix} 0 & 1 & 1 \\ 1 & \frac{1}{2} & 1 \end{matrix} \right| \begin{matrix} 0 \\ 0 \end{matrix} \Bigg|$$

$$\begin{cases} 0 + x_2 + x_3 = 0 \\ x_1 + \frac{1}{2} x_2 + x_3 = 0 \end{cases} ==> \begin{cases} x_1 = -\frac{1}{2} x_3 \\ x_2 = -x_3 \end{cases}$$

$$x_1 = -\frac{1}{2} x_3$$
$$x_2 = -x_3$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} x_3 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -\frac{1}{2} \\ -1 \\ 1 \end{bmatrix}, x_3 \neq 0.$$

$\lambda = 4$:

$$\left| \begin{matrix} 2-4 & 1 & 1 \\ 2 & 3-4 & 2 \\ -2 & 1 & 2-4 \end{matrix} \right| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \Bigg| = 0$$

$$\left| \begin{matrix} -2 & 1 & 1 \\ 2 & -1 & 2 \\ -2 & 1 & -2 \end{matrix} \right| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \Bigg| = \left| \begin{matrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & 1 \\ 1 & -\frac{1}{2} & 1 \end{matrix} \right| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \Bigg| = \left| \begin{matrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & 1 \end{matrix} \right| \begin{matrix} 0 \\ 0 \end{matrix} \Bigg|$$

$$\begin{cases} x_1 - \frac{1}{2} x_2 - \frac{1}{2} x_3 = 0 \\ x_1 - \frac{1}{2} x_2 + x_3 = 0 \end{cases} ==> \begin{cases} x_1 = \frac{1}{2} x_2 \\ x_3 = 0 \end{cases}$$

$$x_1 = \frac{1}{2} x_2$$
$$x_3 = 0$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x_2 \\ x_2 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix}, x_2 \neq 0.$$

If $\lambda = 2$ then

$$b = \begin{bmatrix} -\frac{1}{2}x_3 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -\frac{1}{2} \\ -1 \\ 1 \end{bmatrix}, x_3 \neq 0$$

Answer:

$$\lambda = 1: \ a = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix}$$

$$\lambda = 2 \quad b = \begin{bmatrix} -\frac{1}{2}x_3 \\ -x_3 \\ x_3 \end{bmatrix}$$

$$\lambda = 4 \quad c = \begin{bmatrix} \frac{1}{2}x_2 \\ x_2 \\ 0 \end{bmatrix}$$

If $\lambda = 2$ then $X = \begin{bmatrix} -\frac{1}{2}x_3 \\ -x_3 \\ x_3 \end{bmatrix}$

3. If A.x=λx, where A= ||||211232−212||||, determine the eigen values of the matrix A, and an eigen vector corresponding to each eigen value.
   If
   λ=4
   ,what is c

Solution

$$\hat{A}x = \lambda x$$

$$\hat{A}x = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ -2 & 1 & 2 \end{bmatrix}$$

Let’s determine the eigen values of matrix A

$$\hat{A}x = \lambda\hat{E}x$$

$$\hat{A}x - \lambda\hat{E}x = 0$$

$$\left(\hat{A} - \lambda\hat{E}\right)x = 0$$

$$det\begin{bmatrix} 2-\lambda & 1 & 1 \\ 2 & 3-\lambda & 2 \\ -2 & 1 & 2-\lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 2 & 3-\lambda & 2 \\ -2 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\cdot(3-\lambda)\cdot(2-\lambda) + 1\cdot 2\cdot(-2) + 2\cdot 1\cdot 1 - (-2)\cdot(3-\lambda)\cdot 1 - 1\cdot 2\cdot(2-\lambda) - 1\cdot 2 \cdot(2-\lambda) = 0$$

$$(2-\lambda)^2\cdot(3-\lambda) - 4 + 2 + 6 - 2\lambda + 4\lambda - 8 = 0$$

$$(2-\lambda)^2\cdot(3-\lambda) + 2\lambda - 4 = 0$$

$$(2-\lambda)^2\cdot(3-\lambda) - 2(2-\lambda) = 0$$

$$(2-\lambda)\cdot\big((2-\lambda)\cdot(3-\lambda) - 2\big) = 0$$

$$(2-\lambda)\cdot(\lambda^2 - 5\lambda + 6 - 2) = 0$$

$$(2-\lambda)\cdot(\lambda^2 - 5\lambda + 4) = 0$$

$$\lambda_1 = 2$$

$$\lambda^2 - 5\lambda + 4 = 0$$

$$D = 25 - 16 = 9$$

$$\lambda_2 = 4 \;\; \lambda_3 = 1$$

$$\lambda = 1, \lambda = 2, \lambda = 4.$$

Determine the eigen vector corresponding to each eigen value

$\lambda = 1$:

$$\left|\begin{matrix} 2-1 & 1 & 1 \\ 2 & 3-1 & 2 \\ -2 & 1 & 2-1 \end{matrix}\middle|\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right| = 0$$

$$\left|\begin{matrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -2 & 1 & 1 \end{matrix}\middle|\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right| = \left|\begin{matrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 3 & 3 \end{matrix}\middle|\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}\right| = \left|\begin{matrix} 1 & 1 & 1 \\ 0 & 3 & 3 \end{matrix}\middle|\begin{matrix} 0 \\ 0 \end{matrix}\right|$$

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ 0 + x_2 + x_3 = 0 \end{cases} ==> \begin{cases} x_1 \qquad\qquad = 0 \\ \qquad x_2 + x_3 = 0 \end{cases}$$

$$x_1 = 0$$
$$x_2 = -x_3$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, x_3 \neq 0.$$

$\lambda = 2$:

$$\left| \begin{matrix} 2-2 & 1 & 1 \\ 2 & 3-2 & 2 \\ -2 & 1 & 2-2 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right| = 0$$

$$\left| \begin{matrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ -2 & 1 & 0 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right| = \left| \begin{matrix} 0 & 1 & 1 \\ 1 & \frac{1}{2} & 1 \\ 0 & 2 & 2 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right| = \left| \begin{matrix} 0 & 1 & 1 \\ 1 & \frac{1}{2} & 1 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \end{matrix} \right|$$

$$\begin{cases} 0 + x_2 + x_3 = 0 \\ x_1 + \frac{1}{2}x_2 + x_3 = 0 \end{cases} ==> \begin{cases} x_1 = -\frac{1}{2}x_3 \\ x_2 = -x_3 \end{cases}$$

$$x_1 = -\frac{1}{2}x_3$$
$$x_2 = -x_3$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}x_3 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -\frac{1}{2} \\ -1 \\ 1 \end{bmatrix}$$

$\lambda = 4$:

$$\left| \begin{matrix} 2-4 & 1 & 1 \\ 2 & 3-4 & 2 \\ -2 & 1 & 2-4 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right| = 0$$

$$\left| \begin{matrix} -2 & 1 & 1 \\ 2 & -1 & 2 \\ -2 & 1 & -2 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right| = \left| \begin{matrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & 1 \\ 1 & -\frac{1}{2} & 1 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right| = \left| \begin{matrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & 1 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \end{matrix} \right|$$

$$\begin{cases} x_1 - \frac{1}{2}x_2 - \frac{1}{2}x_3 = 0 \\ x_1 - \frac{1}{2}x_2 + x_3 = 0 \end{cases} ==> \begin{cases} x_1 = \frac{1}{2}x_2 \\ x_3 = 0 \end{cases}$$

$$x_1 = \frac{1}{2}x_2$$
$$x_3 = 0$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x_2 \\ x_2 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix}, x_2 \neq 0.$$

Answer:

$$\lambda = 1:\ a = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix}$$

$$\lambda = 2 \quad b = \begin{bmatrix} -\frac{1}{2}x_3 \\ -x_3 \\ x_3 \end{bmatrix}$$

$$\lambda = 4 \quad c = \begin{bmatrix} \frac{1}{2}x_2 \\ x_2 \\ 0 \end{bmatrix}$$

4. Solve the set of linear equations by the matrix method: a+3b+2c=3, 2a-b-3c= -8, 5a+2b+c=9. Solve for c

Solution

$$\begin{cases} a + 3b + c = 3 \\ 2a - b - 3c = -8 \\ 5a + 2b + c = 9 \end{cases}$$

$$\left|\begin{matrix} 1 & 3 & 1 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{matrix}\right| \begin{matrix} 3 \\ -8 \\ 9 \end{matrix}\right|$$

Using Cramer's rule

$$\Delta = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{vmatrix} =$$

$$= 1 \cdot (-1) \cdot 1 + 3 \cdot (-3) \cdot 5 + 2 \cdot 2 \cdot 1 - 5 \cdot (-1) \cdot 1 - 1 \cdot 2 \cdot (-3) - 2 \cdot 3 \cdot 1 =$$

$$= -1 - 45 + 4 + 5 + 6 - 6 = -37$$

$$\Delta_a = \begin{vmatrix} 3 & 3 & 1 \\ -8 & -1 & -3 \\ 9 & 2 & 1 \end{vmatrix} =$$

$$= 3 \cdot (-1) \cdot 1 + 3 \cdot (-3) \cdot 9 + (-8) \cdot 2 \cdot 1 - 9 \cdot (-1) \cdot 1 - 3 \cdot 2 \cdot (-3) - (-8) \cdot 3 \cdot 1 =$$

$$= -3 - 81 - 16 + 9 + 18 + 24 = -49$$

$$\Delta_b = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -8 & -3 \\ 5 & 9 & 1 \end{vmatrix} =$$

$$= 1 \cdot (-8) \cdot 1 + 3 \cdot (-3) \cdot 5 + 2 \cdot 9 \cdot 1 - 5 \cdot (-8) \cdot 1 - 1 \cdot 9 \cdot (-3) - 2 \cdot 3 \cdot 1 =$$

$$= -8 - 45 + 18 + 40 + 27 - 6 = 26$$

$$\Delta_c = \begin{vmatrix} 1 & 3 & 3 \\ 2 & -1 & -8 \\ 5 & 2 & 9 \end{vmatrix} =$$

$$= 1 \cdot (-1) \cdot 9 + 3 \cdot (-8) \cdot 5 + 2 \cdot 2 \cdot 3 - 5 \cdot (-1) \cdot 3 - 1 \cdot 2 \cdot (-8) - 2 \cdot 3 \cdot 9 =$$

$$= -9 - 120 + 12 + 15 + 16 - 54 = -140$$

$$a = \frac{\Delta_a}{\Delta} = \frac{-49}{-37} = 1\frac{12}{37}$$

$$b = \frac{\Delta_b}{\Delta} = \frac{26}{-37}$$

$$c = \frac{\Delta_c}{\Delta} = \frac{-140}{-37} = 3\frac{29}{37}$$

$a = 1\frac{12}{37}$, $b = \frac{26}{-37}$, $c = 3\frac{29}{37}$

Answer: $c = 3\frac{29}{37}$.

5. If A.x=λx, where A= ||||211232−212||||, determine the eigen values of the matrix A, and an eigen vector corresponding to each eigen value. If λ=1

If
λ=1
,what is a

Solution

$$\hat{A}x = \lambda x$$

$$\hat{A}x = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ -2 & 1 & 2 \end{bmatrix}$$

Let’s determine the eigen values of matrix A

$$\hat{A}x = \lambda\hat{E}x$$

$$\hat{A}x - \lambda\hat{E}x = 0$$

$$(\hat{A} - \lambda\hat{E})x = 0$$

$$det\begin{bmatrix} 2-\lambda & 1 & 1 \\ 2 & 3-\lambda & 2 \\ -2 & 1 & 2-\lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 2 & 3-\lambda & 2 \\ -2 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\cdot(3-\lambda)\cdot(2-\lambda) + 1\cdot 2\cdot(-2) + 2\cdot 1\cdot 1 - (-2)\cdot(3-\lambda)\cdot 1 - 1\cdot 2\cdot(2-\lambda) - 1\cdot 2 \cdot (2-\lambda) = 0$$

$$(2-\lambda)^2\cdot(3-\lambda) - 4 + 2 + 6 - 2\lambda + 4\lambda - 8 = 0$$

$$(2-\lambda)^2\cdot(3-\lambda) + 2\lambda - 4 = 0$$

$$(2-\lambda)^2\cdot(3-\lambda) - 2(2-\lambda) = 0$$

$$(2-\lambda)\cdot\big((2-\lambda)\cdot(3-\lambda) - 2\big) = 0$$

$$(2-\lambda)\cdot(\lambda^2 - 5\lambda + 6 - 2) = 0$$

$$(2-\lambda)\cdot(\lambda^2 - 5\lambda + 4) = 0$$

$$\lambda_1 = 2$$

$$\lambda^2 - 5\lambda + 4 = 0$$

$$D = 25 - 16 = 9$$

$$\lambda_2 = 4 \ \ \lambda_3 = 1$$

$$\lambda = 1, \lambda = 2, \lambda = 4$$

Determine the eigen vector corresponding to each eigen value.

$\lambda = 1$:

$$\begin{vmatrix} 2-1 & 1 & 1 \\ 2 & 3-1 & 2 \\ -2 & 1 & 2-1 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -2 & 1 & 1 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 3 & 3 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 3 & 3 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ 0 + x_2 + x_3 = 0 \end{cases} ==> \begin{cases} x_1 \qquad\qquad = 0 \\ \qquad x_2 + x_3 = 0 \end{cases}$$

$$x_1 = 0$$
$$x_2 = -x_3$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, x_3 \neq 0.$$

$\lambda = 2$:

$$\begin{vmatrix} 2-2 & 1 & 1 \\ 2 & 3-2 & 2 \\ -2 & 1 & 2-2 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 & 1 & 1 \\ 2 & 1 & 2 \\ -2 & 1 & 0 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 \\ 1 & \frac{1}{2} & 1 \\ 0 & 2 & 2 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 \\ 1 & \frac{1}{2} & 1 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\begin{cases} 0 + x_2 + x_3 = 0 \\ x_1 + \frac{1}{2}x_2 + x_3 = 0 \end{cases} ==> \begin{cases} x_1 = -\frac{1}{2}x_3 \\ x_2 = -x_3 \end{cases}$$

$$x_1 = -\frac{1}{2}x_3$$
$$x_2 = -x_3$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}x_3 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -\frac{1}{2} \\ -1 \\ 1 \end{bmatrix}$$

$\lambda = 4$:

$$\begin{vmatrix} 2-4 & 1 & 1 \\ 2 & 3-4 & 2 \\ -2 & 1 & 2-4 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = 0$$

$$\begin{vmatrix} -2 & 1 & 1 \\ 2 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & 1 \\ 1 & -\frac{1}{2} & 1 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 1 & -\frac{1}{2} & 1 \end{vmatrix}\begin{vmatrix} 0 \\ 0 \end{vmatrix}$$

$$\begin{cases} x_1 - \frac{1}{2}x_2 - \frac{1}{2}x_3 = 0 \\ x_1 - \frac{1}{2}x_2 + x_3 = 0 \end{cases} ==> \begin{cases} x_1 = \frac{1}{2}x_2 \\ x_3 = 0 \end{cases}$$

$$x_1 = \frac{1}{2}x_2$$
$$x_3 = 0$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x_2 \\ x_2 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix}, x_2 \neq 0.$$

If $\lambda = 1$ then

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, x_3 \neq 0.$$

Answer:

$$\lambda = 1: \ a = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix}$$

$$\lambda = 2 \quad b = \begin{bmatrix} -\frac{1}{2}x_3 \\ -x_3 \\ x_3 \end{bmatrix}$$

$$\lambda = 4 \quad c = \begin{bmatrix} \frac{1}{2}x_2 \\ x_2 \\ 0 \end{bmatrix}$$

If $\lambda = 1$ then $a = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix}$