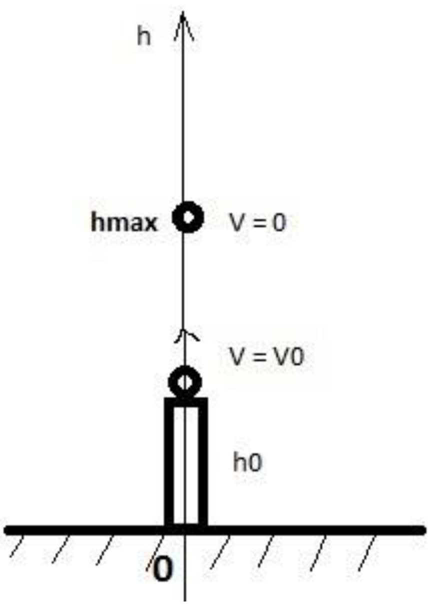
Answer on Question #55823 – Math – Calculus

4. A tennis ball machine serves a ball vertically into the air from a height of 2 feet, with an initial speed of 120 feet per second. What is the maximum height, in feet, the ball will attain? Round to the nearest whole foot.

Solution



$h(t)$ is height, $v(t) = \dot{h}(t)$ is velocity, $a(t) = \dot{v}(t) = \ddot{h}(t)$ is acceleration

$$v0 = 120\frac{feet}{s}, \qquad g = 9{,}8\frac{m}{s^2} = 9{,}8 * 3{,}28\left(\frac{feet}{s^2}\right) = 32{,}14\frac{feet}{s^2}$$

As we know $a(t) = const = -g$, so $v(t) = \int a(t)dt = v_0 - gt$ (v_0 is the initial velocity)

Also $h(t) = \int v(t)\,dt = \int(v_0 - gt)\,dt = h_0 + v_0 t - \frac{gt^2}{2}$ (h_0 is the initial height)

$h(t_{max}) = h_{max}$ when $\dot{h}(t\_max) = v(t\_max) = 0$, so $0 = v_0 - gt_{max}, t_{max} = \frac{v_0}{g}$,

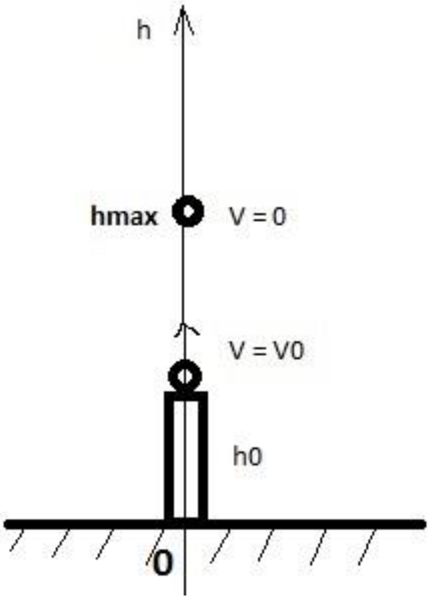
$h_{max} = h(t_{max}) = h_0 + v_0\,t - \left(\frac{g}{2}\right)\left(\frac{v_0}{g}\right)^2 = h_0 + \frac{v_0^2}{2g}$ (you can get this much easier, if you know The Law of Conservation of Energy: $mv^2 + mg\Delta h = const$, so in this case $mv^2 = mg\Delta h, \Delta h = \frac{v^2}{2g}$)

$$h_{max} = 2 + \frac{120^2}{2 * 32{,}14} = 226{,}02\ (feet)$$

Answer: 226 feet

5. A tennis ball machine serves a ball vertically into the air from a height of 2 feet, with an initial speed of 110 feet per second. After how many seconds does the ball attain its maximum height? Round to the nearest hundredth.

Solution:



$h(t)$ is height, $v(t) = \dot{h}(t)$ is velocity, $a(t) = \dot{v}(t) = \ddot{h}(t)$ is acceleration

$$v0 = 110\frac{feet}{s}, \qquad g = 9{,}8\frac{m}{s^2} = 9{,}8 * 3{,}28\left(\frac{feet}{s^2}\right) = 32{,}14\frac{feet}{s^2}$$

As we know $a(t) = const = -g$, so $v(t) = \int a(t)dt = v_0 - gt$ (v_0 is the initial condition)

Also $h(t) = \int v(t)\,dt = \int (v_0 - gt)\,dt = h_0 + v_0 t - \frac{gt^2}{2}$

$h(t_{max}) = h_{max}$ when $\dot{h}(t_{max}) = v(t_{max}) = 0$, so $0 = v_0 - gt_{max}$, $t_{max} = \frac{v_0}{g}$,

$$t_{max} = \frac{110}{32{,}14} = 3{,}4225\ (s)$$

Answer: 3,42 s.

6. The finishing time for a runner completing the 200-meter dash is affected by the tail-wind speed, s . The change, t, in a runner's performance is modeled by the function shown below:

$$t = 0.0119s^2 - 0.308s - 0.0003$$

Predict the change in a runner's finishing time with a wind speed of 5 meters/second. Note: A negative answer means the runner finishes with a lower time. Round to the nearest hundredths.

Solution:

$$t(s) = 0.0119s^2 - 0.308s - 0.0003$$

$$t(5) = 0.0119 * (5^2) - 0.308 * (5) - 0.0003 = -1{,}2428\ (s)$$

Answer: -1,24 s, the runner finishes with a lower time.