

Answer on Question #55822 – Math – Algebra

1. A rectangle's width is 6 feet less than its length. Write a quadratic function that expresses the rectangle's area in terms of its length.

A: $A(L) = L^2 - 6L$

B: $A(L) = L^2 + 6L$

C: $A(L) = LW$

D: $A(L) = L - 6$

Solution

Area of a rectangle is equal to the product of its length and width,

L = length, W = width, A = rectangle's area;

$A = L \cdot W$, where $W = L - 6$,

$$A(L) = L \cdot (L - 6) = L^2 - 6L$$

Answer: A: $A(L) = L^2 - 6L$.

2. A rectangle's width is 5 feet less than its length. If the area of the rectangle is 546 square feet, what is its length, in feet?

Solution

Area of a rectangle is equal to the product of its length and width,

L = length, W = width and A = rectangles area;

$A = L \cdot W$, where $W = L - 5$ feet and $A = 546$ square feet

$$A(L) = L \cdot (L - 5) = L^2 - 5L = 546$$

We obtained an equation with an unknown L :

$$L^2 - 5L = 546$$

$$L^2 - 5L - 546 = 0$$

$$a = 1, \quad b = -5, \quad c = -546$$

Discriminant is

$$D = b^2 - 4 \cdot a \cdot c = (-5)^2 - 4 \cdot 1 \cdot (-546) = 25 + 2184 = 2209$$

Solutions of the equation are

$$L_{1,2} = \frac{-b \pm \sqrt{D}}{2 \cdot a},$$

Take the value, where $L > 0$, because length can't have a negative sign:

$$L_1 = \frac{-b - \sqrt{D}}{2} = \frac{-(-5) - \sqrt{2209}}{2} = \frac{5 - 47}{2} = \frac{-42}{2} = -21 < 0,$$

$$L_2 = \frac{-b + \sqrt{D}}{2} = \frac{-(-5) + \sqrt{2209}}{2} = \frac{5 + 47}{2} = \frac{52}{2} = 26 \text{ feet.}$$

$L = 26$ feet

Answer: $L = 26$ feet.

3. A tennis ball machine serves a ball vertically into the air from the height of 2 feet, with an initial speed of 110 feet per second. Which equation correctly models the ball's height as a function of time?

A: $h(t) = -16t^2 - 2t + 110$

B: $h(t) = -16t^2 + 110t + 2$

C: $h(t) = -16t^2 - 110t + 2$

D: $h(t) = -16t^2 + 2t + 110$

Solution

Let's consider the equation of motion of the ball

$$h(t) = h_0 + v_0 t + \frac{at^2}{2}$$

where h_0 and v_0 are initial values for the height and speed, respectively.

$h_0 = 2$ feet, $v_0 = 110$ feet per second (f/s)

also we have accelerated downward $a < 0$: $a/2 = -16$

$$h(t) = 2 + 110t - 16t^2 = -16t^2 + 110t + 2$$

Answer: B: $h(t) = -16t^2 + 110t + 2$