Answer on Question #55791 – Math – Calculus

2. If f(x,y)=tan^ −1 y/x, find f x.

3. Suppose f(x,y) =x^3 y^ 2 –sin^ 2xcos2y, what is ∂f/∂y?

4. Suppose f(x,y)=sin^2xcosy+xy^ 2, what is ∂f/∂x ?

5. If f(x,y)=x^ 2 y^ 3 –2y^ −2, find f y

6. If f(x,y)=4x^3 –3y^2, find f x

Solution

2. $\mathrm{f(x,y)} = tan^{-1}\, y/x$

If tan^{-1} means the inverse of the tangent, then the answer will be

$$\mathrm{f_x} = \frac{\partial \mathrm{f(x,y)}}{\partial \mathrm{x}} = (\tan^{-1}(\mathrm{y/x}))'_{\mathrm{x}} = (\arctan(\mathrm{y/x}))'_{\mathrm{x}} = \frac{1}{1+\left(\frac{y}{x}\right)^2} * \left(\frac{y}{x}\right)'_x = \frac{1}{1+\left(\frac{y}{x}\right)^2} * \left(-\frac{y}{x^2}\right)$$

$$\mathrm{f_x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} * \left(-\frac{y}{x^2}\right) = -\frac{y}{x^2\left(1+\left(\frac{y}{x}\right)^2\right)} = -\frac{y}{x^2+y^2},$$

because $\frac{\partial(\arctan(\mathrm{t}))}{\partial \mathrm{t}} = \frac{1}{1+t^2}, \frac{\partial(x^{\mathrm{n}})}{\partial \mathrm{x}} = nx^{\mathrm{n}-1},$ where n is integer, and the chain rule of differentiation was applied.

If tan^{-1} means the tangent raised to the power of (-1), then the answer will be

$$\mathrm{f_x} = \frac{\partial \mathrm{f(x,y)}}{\partial \mathrm{x}} = \left(\tan^{-1}(\mathrm{y/x})\right)'_{\mathrm{x}} = -tan^{-2}\left(\frac{\mathrm{y}}{\mathrm{x}}\right)\frac{1}{cos^2\left(\frac{y}{x}\right)}((\mathrm{y/x}))'_{\mathrm{x}} = -\frac{1}{tan^2\left(\frac{y}{x}\right)} \cdot \frac{1}{cos^2\left(\frac{y}{x}\right)} \cdot \left(-\frac{y}{x^2}\right) =$$

$= \frac{y}{x^2 sin^2(y/x)},$ because $\frac{\partial(x^{\mathrm{n}})}{\partial \mathrm{x}} = nx^{\mathrm{n}-1},$ where n is integer, $\frac{\partial(\tan(\mathrm{t}))}{\partial \mathrm{t}} = \frac{1}{cos^2(t)},$ and the chain rule of differentiation was applied

3. $f(x,y) = x^3y^2 - sin^2(x)\cos(2y)$

$$\frac{\partial \mathrm{f}}{\partial \mathrm{y}} = x^3 \cdot 2y - sin^2(x) \cdot (-\sin(2y) \cdot 2) = 2x^3y + 2sin^2(x)\sin(2y),$$

because $\frac{\partial(\mathrm{p-q})}{\partial \mathrm{y}} = \frac{\partial(\mathrm{p})}{\partial \mathrm{y}} - \frac{\partial(\mathrm{q})}{\partial \mathrm{y}}$, $\frac{\partial(\mathrm{g(x)h(y)})}{\partial \mathrm{y}} = g(x)\frac{\partial(\mathrm{h(y)})}{\partial \mathrm{y}}$, $\frac{\partial(y^{\mathrm{n}})}{\partial \mathrm{y}} = ny^{\mathrm{n}-1}$, where n is integer,

$\frac{\partial(\mathrm{ah(y)})}{\partial \mathrm{y}} = a\frac{\partial(\mathrm{h(y)})}{\partial \mathrm{y}}$, where a is a constant,

$\frac{\partial(\cos(\mathrm{t}))}{\partial \mathrm{t}} = -\sin(t),$

and the chain rule of differentiation was applied.

4. $f(x,y) = sin^2(x)\cos(y) + xy^2$

$\frac{\partial \mathrm{f}}{\partial \mathrm{x}} = 2\sin(x)\frac{\partial \sin(\mathrm{x})}{\partial \mathrm{x}}\cos(y) + y^2 = 2\sin(x)\cos(\mathrm{x})\cos(\mathrm{y}) + y^2 = \sin(2x)\cos(y) + y^2$, because $\frac{\partial(\mathrm{p+q})}{\partial \mathrm{x}} = \frac{\partial(\mathrm{p})}{\partial \mathrm{x}} + \frac{\partial(\mathrm{q})}{\partial \mathrm{x}}$, $\frac{\partial(\mathrm{g(x)h(y)})}{\partial \mathrm{x}} = h(y)\frac{\partial(\mathrm{g(x)})}{\partial \mathrm{x}}$, $\frac{\partial(x^{\mathrm{n}})}{\partial \mathrm{x}} = nx^{\mathrm{n}-1}$, where n is integer, $\frac{\partial(\sin(\mathrm{t}))}{\partial \mathrm{t}} = \cos(t)$, and the chain rule of differentiation was applied.

5. $f(x,y) = x^2y^3 - 2y^{-2}$

$\mathrm{f_y} = x^2 \cdot (3y^2) - 2 \cdot (-2) \cdot y^{-3} = 3x^2y^2 + 4y^{-3}$, because $\frac{\partial(\mathrm{p-q})}{\partial \mathrm{y}} = \frac{\partial(\mathrm{p})}{\partial \mathrm{y}} - \frac{\partial(\mathrm{q})}{\partial \mathrm{y}}$,

$\frac{\partial(\mathrm{g(x)h(y)})}{\partial \mathrm{y}} = g(x)\frac{\partial(\mathrm{h(y)})}{\partial \mathrm{y}}$, $\frac{\partial(\mathrm{ah(y)})}{\partial \mathrm{y}} = a\frac{\partial(\mathrm{h(y)})}{\partial \mathrm{y}}$, where a is a constant, $\frac{\partial(y^{\mathrm{n}})}{\partial \mathrm{y}} = ny^{\mathrm{n}-1}$, where n is integer,

6. $f(x,y) = 4x^3 - 3y^2$

$\mathrm{f_x} = 4 \cdot 3 \cdot x^2 = 12x^2$, because $\frac{\partial(\mathrm{p}-\mathrm{q})}{\partial \mathrm{x}} = \frac{\partial(\mathrm{p})}{\partial \mathrm{x}} - \frac{\partial(\mathrm{q})}{\partial \mathrm{x}}$, $\frac{\partial(\mathrm{ag(x)})}{\partial \mathrm{x}} = a\frac{\partial(\mathrm{g(x)})}{\partial \mathrm{x}}$, where a is a constant, $\frac{\partial(x^{\mathrm{n}})}{\partial \mathrm{x}} = nx^{\mathrm{n}-1}$, $\frac{\partial(y^2)}{\partial \mathrm{x}} = 0$, where n is integer.

Answer:

2. $\mathrm{f_x} = -\frac{y}{x^2+y^2}$

3. $\frac{\partial \mathrm{f}}{\partial \mathrm{y}} = 2x^3y + 2sin^2(x)\sin(2y)$

4. $\frac{\partial \mathrm{f}}{\partial \mathrm{x}} = \sin(2x)\cos(y) + y^2$

5. $\mathrm{f_y} = 3x^2y^2 + 4y^{-3}$

6. $\mathrm{f_x} = 12x^2$