Answer on Question #55751 – Math – Linear Algebra

6.Solve the set of linear equations by Guassian elimination method : a+2b+3c=5, 3a-b+2c=8, 4a-6b-4c=-2. Find c
4;5;9;10

Solution

a+2b+3c=5, 3a-b+2c=8, 4a-6b-4c=-2

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 3 & -1 & 2 & 8 \\ 4 & -6 & -4 & -2 \end{array}\right) \xrightarrow{1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & \frac{7}{3} & \frac{7}{3} & \frac{7}{3} \\ 0 & \frac{7}{2} & 4 & \frac{22}{4} \end{array}\right) \xrightarrow{2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{4}{2} \end{array}\right)$$

1) The second and the third rows are multiplied by -1/3 and -1/4 respectively and added to the first line
2) The second row is multiplied by 3/2 and the third line is subtracted from it
So, we get that: $-\frac{1}{2}c = -\frac{4}{2} \to c = 4$
Answer: c = 4

7.Solve the set of linear equations by the matrix method : a+3b+2c=3 , 2a-b-3c= -8, 5a+2b+c=9. Sove for b
9;-3;5;-4

Solution
Method 1 (matrix method)

Let

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{pmatrix}, X = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, B = \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix}$$

The solution of the system of equations is sought in the form

$X = A^{-1} * B$

Let's find A^{-1}

The determinant of the system is

$$\Delta = \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{vmatrix} = 1\begin{vmatrix} -1 & -3 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} = 1(-1+6) - 3(2+15) + 2(4+5)$$

$$\Delta = 5 - 51 + 18 = -28$$

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & -3 \\ 2 & 1 \end{vmatrix} = -1 + 6 = 5, \quad A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} = -(2+15) = -17,$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} = (4+5) = 9,$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = -(3-4) = -(-1) = 1, \; A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} = (1-10) = -9,$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix} = -(2-15) = -(-13) = 13,$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 3 & 2 \\ -1 & -3 \end{vmatrix} = -9 + 2 = -7, \; A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 2 \\ -1 & -3 \end{vmatrix} = -(-9+2) = 7,$$

$$A_{33} = (-1)^{3+1}\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -1 - 6 = -7$$

$$\tilde{A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^T = \begin{pmatrix} 5 & -17 & 9 \\ 1 & -9 & 13 \\ -7 & 7 & -7 \end{pmatrix}^T = \begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}$$

The inverse of A is

$$A^{-1} = \frac{1}{\Delta}\tilde{A} = -\frac{1}{28}\begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = -\frac{1}{28}\begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}\begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix}$$

$$b = -\frac{1}{28} * (-17 * 3 - 9 \cdot (-8) + 7 * 9) = -\frac{84}{28} = -3$$

Method 2 (Gaussian elimination method)

a+3b+2c=3 , 2a-b-3c= -8, 5a+2b+c=9

$$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 3 \\ 2 & -1 & -3 & -8 \\ 5 & 2 & 1 & 9 \end{array}\right) \xrightarrow{1} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 3 \\ 0 & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ 0 & \frac{13}{5} & \frac{9}{5} & \frac{6}{5} \end{array}\right) \xrightarrow{2} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 3 \\ 0 & \frac{13}{5} & \frac{13}{5} & \frac{26}{5} \\ 0 & 0 & \frac{4}{5} & \frac{20}{5} \end{array}\right)$$

1) The second and the third rows are multiplied by -1/2 and -1/5 respectively and added to the first row
2) The second row is multiplied by 26/35 and the third row is subtracted from it
So, we get that: $\frac{4}{5}c = \frac{20}{5} \to c = 5$
From the second row we get that: $\frac{13}{5}b + \frac{13}{5}c = \frac{26}{5} \to \frac{13}{5}b + \frac{13}{5}5 = \frac{26}{5} \to \frac{13}{5}b + 13 = \frac{26}{5} \to b = -3.$
Answer: b = -3

8.Solve the set of linear equations by Guassian elimination method : a+2b+3c=5, 3a-b+2c=8, 4a-6b-4c=-2. Find b
4;-5;-3;5

Solution

a+2b+3c=5, 3a-b+2c=8, 4a-6b-4c=-2
From part 6 we got that c = 4.
From the second row we get that $\frac{7}{2}b + \frac{7}{2}c = \frac{7}{2} \to \frac{7}{2}b + \frac{7}{2} * 4 = \frac{7}{2} \to \frac{7}{2}b + 14 = \frac{7}{2} \to b = -3.$
Answer: b = -3

9.Solve the set of linear equations by Guassian elimination method : a+2b+3c=5, 3a-b+2c=8, 4a-6b-4c=-2. Find a
-1;4;5;-11

Solution

a+2b+3c=5, 3a-b+2c=8, 4a-6b-4c=-2
From part 8 we get that b = -3, c = 4.

From the first row of $\left(\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & \frac{7}{2} & \frac{7}{2} & \frac{7}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{4}{2} \end{array}\right)$ we get that

$1a + 2b + 3c = 5 \to a + 2 * (-3) + 3 * 4 = 5 \to a - 6 + 12 = 5 \to a = -1.$

Answer: a = -1.

10. Solve the set of linear equations by the matrix method : a+3b+2c=3 , 2a-b-3c= -8, 5a+2b+c=9. Sove for a 2;4;7;3

Solution

a+3b+2c=3, 2a-b-3c= -8, 5a+2b+c=9

Let

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{pmatrix}, X = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, B = \begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix}$$

The solution of the system of equations is sought in the form

$X = A^{-1} * B$

Let's find A^{-1}.

The determinant of the system is

$$\Delta = \begin{vmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{vmatrix} = 1\begin{vmatrix} -1 & -3 \\ 2 & 1 \end{vmatrix} - 3\begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} = 1(-1+6) - 3(2+15) + 2(4+5)$$

$\Delta = 5 - 51 + 18 = -28$

$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & -3 \\ 2 & 1 \end{vmatrix} = -1 + 6 = 5, \quad A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} = -(2 + 15) = -17,$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} = (4 + 5) = 9,$

$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = -(3 - 4) = -(-1) = 1, \; A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} = (1 - 10) = -9,$

$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix} = -(2 - 15) = -(-13) = 13,$

$A_{31} = (-1)^{3+1}\begin{vmatrix} 3 & 2 \\ -1 & -3 \end{vmatrix} = -9 + 2 = -7, \; A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 2 \\ -1 & -3 \end{vmatrix} = -(-9 + 2) = 7,$

$A_{33} = (-1)^{3+1}\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -1 - 6 = -7$

$$\tilde{A} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^T = \begin{pmatrix} 5 & -17 & 9 \\ 1 & -9 & 13 \\ -7 & 7 & -7 \end{pmatrix}^T = \begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}$$

$$\tilde{A} = \begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}$$

The inverse of A is

$$A^{-1} = \frac{1}{\Delta}\tilde{A} = -\frac{1}{28}\begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = -\frac{1}{28}\begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}\begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix}$$

$a = -\frac{1}{28} * (5 * 3 - 8 - 7 * 9) = \frac{56}{28} = 2.$

Answer: a = 2.