Answer on Question #55750 – Math – Linear Algebra

1. Solve the equations 5x + 2y = 14, 3x - 4y = 24.
x = 4, y = -4
x = 4, y = -2
x = 4, y = -3
x = 4, y = 3

Solution

Multiply the first equation of the system by 2:

$$\begin{cases} 5x + 2y = 14 \mid *2 \\ 3x - 4y = 24 \end{cases}$$

Add two equations

$$+\begin{cases} 10x + 4y = 28 \\ 3x - 4y = 24 \end{cases}$$

$13x=52.$

Divide both sides of the equation by 13:
$x=4.$

Substitute for x=4 into the first equation of the initial system:
$5*4+2y=14.$

Collect similar terms:
$2y=14-20.$

Simplify:
$2y=-6.$

Divide both sides by 2:
$y=-3.$

Answer: x = 4, y = -3.

2. Solve the linear equation 2x+3y=1, 5x+7y=3.
x=2, y=-1
x=4, y=-2
x=2, y=-2
x=5, y=-3

Solution

Multiply the first equation of the system by 5 and the second equation by (-2)

$$\begin{cases} 2x+3y=1 \mid *5 \\ 5x+7y=3. \mid *(-2) \end{cases}$$

Add two equations:

$$+\begin{cases} 10x+15y=5 \\ -10x-14y=-6 \end{cases}$$

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y=-1.

Substitute for y=-1 into the first equation of the initial system of equations:
2x+3*(-1)=1.

Collect similar terms:
2x=4.

Divide both sides by 2:
x=2.

Answer: x=2, y=-1

3. Solve the linear equations 2x+4y=10 and 3x+6y=15.
x=5-2a,y=a
x=5-2a,y=4
x=5,y=a
x=-2a,y=a

Solution

Divide the first equation by 2 and the second equation by 3:

$$\begin{cases} 2x+4y=10 \mid :2 \\ 3x+6y=15 \mid :3 \end{cases}$$

$$\begin{cases} x+2y=5 \\ x+2y=5, \end{cases}$$

We obtain two identical equations, which give the only equation x+2y=5, hence x=5-2y.

if y=a then x= 5-2a

Answer: x=5-2a, y=a

4. Solve the set of linear equations by the matrix method:
a+3b+2c=3 , 2a-b-3c= -8, 5a+2b+c=9. Solve for c
3 1 5 7

Solution

We have the matrix equation
AX=B, where

$$A=\begin{pmatrix} 1 & 3 & 2 \\ 2 & -1 & -3 \\ 5 & 2 & 1 \end{pmatrix};\ X=\begin{pmatrix} a \\ b \\ c \end{pmatrix};\ B=\begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix}$$

Det A= $1\times(-1)\times1+3\times(-3)\times5+2\times2\times2-5\times(-1)\times2-2\times(-3)\times1-1\times2\times3=-28$

$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & -3 \\ 2 & 1 \end{vmatrix} = 5$ $A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} = -17$ $A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} = 9$

$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = 1$ $A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} = -9$ $A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix} = 13$

$A_{31} = (-1)^{3+1}\begin{vmatrix} 3 & 2 \\ -1 & -3 \end{vmatrix} = -7$ $A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 2 \\ 2 & -3 \end{vmatrix} = 7$ $A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -7$

The inverse of A is

$A^{-1} = -\frac{1}{28}\begin{pmatrix} 5 & -17 & 9 \\ 1 & -9 & 13 \\ -7 & 7 & -7 \end{pmatrix}^T = -\frac{1}{28}\begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}$

Using the matrix method,

$X = -\frac{1}{28}\begin{pmatrix} 5 & 1 & -7 \\ -17 & -9 & 7 \\ 9 & 13 & -7 \end{pmatrix}\begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix} = \begin{pmatrix} -5/28 & -1/28 & 1/4 \\ 17/28 & 9/28 & -1/4 \\ -9/28 & -13/28 & 1/4 \end{pmatrix}\begin{pmatrix} 3 \\ -8 \\ 9 \end{pmatrix} = \begin{pmatrix} -15/28 + 8/28 + 9/4 \\ 51/28 - 72/28 - 9/4 \\ -27/28 + 104/28 + 9/4 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$

Thus, a=2; b=-3; c=5.

Answer: c=5

5. Solve the linear equation : 2x+3y=3, x-2y=5 and 3x+2y=7.

x=2 and y=-1

x=3 and y=1

x=3 and y=-1

x=1 and y=-1

Solution

$\begin{cases} 2x+3y=3 \\ x-2y=5 \\ 3x+2y=7 \end{cases}$

Replace the third equation by the sum of the second and the third equations of the system:

$\begin{cases} 2x+3y=3 \\ x-2y=5 \\ 4x=12 \end{cases}$

Divide both sides of the third equation by 3:

x=3

Substitute for x=3 into the first and the second equations of the system:

$\begin{cases} 6+3y=3, \\ 3-2y=5, \\ x=3, \end{cases}$

Replace the first equation by the sum of the first and the second ones:

9+y=8, hence y=8-9, that is, y=-1.

Notice that y=-1 satisfies each equation of the previous system.

Answer: x=3 and y=-1.