Answer on Question #55689 – Math – Statistics and Probability

Question

In 2014 the Department of Social Services reported that 32% of current marriages in Australia were expected to end in divorce.
Find the probability that more than 8 marriages out of a random sample of 20 marriages which were current in 2014 would end in divorce.

Solution

We need to find the following probability: $p(> 8\ divorces)$.

Obviously, more than 8 divorces means either 9 divorces or 10 divorces or 11 divorces and so on and so forth:

$$p(> 8\ divorces) = p(9\ divorces) + p(10\ divorces) + p(11\ divorces) + \cdots +$$
$$+p(20\ divorces).$$

Now, let us find the probability of exactly k divorces.

A divorce happens with a probability $p = 0.32$.

If we have *k* divorces, then we have $(20 - k)$ happy couples not divorced. Assume that couples 1 through *k* have divorced, while others have not. It means that 1st marriage ended with a divorce (with probability p), and the 2nd one (p again), …, while (k+1)th did not end with a divorce (with probability $1 - p$), … These events occured simultaneously, so we should multiply their probabilities: $p^k(1-p)^{n-k}$.

But it could've happened with the other combination of *k* marriages chosen from 20, while there are $\binom{20}{k} = \frac{20!}{k!(20-k)!}$ ways to choose k couples from 20. Thus, we have

$$p(k\ divorces) = \binom{20}{k} p^k (1-p)^{n-k}.$$

Thus,

$$p(> 8\ divorces) = \sum_{k=9}^{20} \binom{20}{k} p^k (1-p)^{n-k}.$$

Substituting $p = 0.32$ we have

$p(> 8\ divorces) = \sum_{k=9}^{20} \binom{20}{k} 0.32^k 0.68^{20-k} \approx 0.1568$ (it was computed by means of Wolfram|Alpha Widgets: Binomial Distribution Calculator).

In Wolfram Mathematica we can also use

functbin[n_]:=BinomialDistribution[n,0.32];

Probability[$9 \le x \le 20, x \approx functbin[20]$]

In Excel 2013 it can be calculated by means of the following expression:

=BINOM.DIST.RANGE(20;0,32;9;20)

In Excel 2010 and Excel 2013 it can be calculated by means of the following expression:

=BINOM.DIST(20;20;0.32;TRUE)-BINOM.DIST(8;20;0,32;TRUE)

In Excel 2000, Excel XP, Excel 2003, Excel 2007, Excel 2010, Excel 2013 it can be calculated by means of the following expression:

=BINOMDIST(20;20;0.32;TRUE)-BINOMDIST(8;20;0,32;TRUE)

The meaning of the functions are the following:

BINOM.DIST.RANGE(n; p; x; y)=the probability there are between *x* and *y* successes (inclusive) in *n* trials where the probability of success on any trial is *p.*

BINOMDIST(x; n; p; TRUE)=cumulative probability distribution F(*x*) value at *x* for the binomial distribution B(n, p), i.e. the probability that there are at most *x* successes in *n* trials where the probability of success on any trial is *p*. Here *x* is a non-negative integer, *n* is a positive integer, $0<p<1$. A value of TRUE returns the cumulative distribution function.

BINOM.DIST is equivalent to BINOMDIST:

BINOM.DIST(number_success; number_trial; p; TRUE).

In R language it can be calculated by means of the following expression:

pbinom(20,20,0.32)-pbinom(8,20,0.32)

Answer: 0.1568.