Answer on Question #55601 – Math – Calculus

Determine if the series converge absolutely, converge conditionally, or diverge. If it is absolutely convergent, find its sum.

a) $\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)}+2^{-n}\right)$;

b) $\sum_{n=1}^{\infty}(\cos(\pi n)\arctan(n))$.

Solution

a)

$$\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)}+2^{-n}\right)=S_1+S_2$$

$$S_1=\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)}\right)=3\sum_{n=1}^{\infty}\left(\frac{1}{n(n+1)}\right)$$

$$S_2=\sum_{n=1}^{\infty}(2^{-n})$$

The series $\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)}\right)$, $\sum_{n=1}^{\infty}2^{-n}$ have a positive terms.

These series are convergent, their sums are the following:

$S_1=3\sum_{n=1}^{\infty}\frac{1}{n(n+1)}=3\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)=3\lim_{N\to\infty}\sum_{n=1}^{N}\left(\frac{1}{n}-\frac{1}{n+1}\right)=3\lim_{N\to\infty}\left(1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\right.$

$\left.+\frac{1}{4}-\cdots-\frac{1}{N-1}+\frac{1}{N-1}-\frac{1}{N}\right)=3\lim_{N\to\infty}\left(1-\frac{1}{N}\right)=3$ is finite.

$S_2=\frac{a_1}{1-q}=\left|a_1=2^{-1}=\frac{1}{2};q=\frac{1}{2}\right|=\frac{\frac{1}{2}}{1-\frac{1}{2}}=1$ is finite.

So S_1 and S_2 are absolutely convergent, hence $\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)}+2^{-n}\right)$ is absolutely convergent too and its sum is

$S=S_1+S_2=3+1=4.$

b)

$$\sum_{n=1}^{\infty}\cos(\pi n)\,arctgn=\sum_{n=1}^{\infty}(-1)^n arctan(n)$$

$$\lim_{n\to\infty}|a_n|=\lim_{n\to\infty}|\arctan(n)|=\frac{\pi}{2}\neq 0$$

Hence, the given series $\sum_{n=1}^{\infty} \cos(\pi n) \arctan(n)$ *is divergent*, because the necessary condition for convergence of series does not hold.