Answer on Question #55481 – Math – Statistics and Probability

The following probabilities for grades in management science have been determined based on past records:

Grade	Probability
A	0.1
B	0.2
C	0.4
D	0.2
F	0.1
	1.00

The grades are assigned on a 4.0 scale, where an A is a 4.0, a B a 3.0, and so on. Determine the expected grade and variance for the course.

Solution

Let γ be a random variable, which takes on values 4, 3, 2, 1, 0 with the following probabilities:

$$P(4) = P(\gamma = 4) = P(grade\ equals\ 4) = 0.1$$

$$P(3) = P(\gamma = 3) = P(grade\ equals\ 3) = 0.2$$

$$P(2) = P(\gamma = 2) = P(grade\ equals\ 2) = 0.4$$

$$P(1) = P(\gamma = 1) = P(grade\ equals\ 1) = 0.2$$

$$P(0) = P(\gamma = 0) = P(grade\ equals\ 0) = 0.1$$

So the expected grade is given by

$$E(\gamma) = \sum_{k=0}^{4} kP(k) = 4 \cdot 0.1 + 3 \cdot 0.2 + 2 \cdot 0.4 + 1 \cdot 0.2 + 0 \cdot 0.1 = 0.4 + 0.6 + 0.8 + 0.2 = 2$$

Method 1

The variance for the course is equal to

$$Var(\gamma) = \sum_{k=0}^{4} (k - E(\gamma))^2 P(k) =$$

$$= (4-2)^2 \cdot 0.1 + (3-2)^2 \cdot 0.2 + (2-2)^2 \cdot 0.4 + (1-2)^2 \cdot 0.2 + (0-2)^2 \cdot 0.1 = 1.2.$$

Method 2

The variance for the course is equal to

$$Var(\gamma) = \sum_{k=0}^{4} k^2 P(k) - (E(\gamma))^2 =$$

$= 4^2 \cdot 0.1 + 3^2 \cdot 0.2 + 2^2 \cdot 0.4 + 1^2 \cdot 0.2 + 0^2 \cdot 0.1 - 2^2 = 16 \cdot 0.1 + 9 \cdot 0.2 + 4 \cdot 0.4 + 0.2 + +0 - 4 = 1.2.$