Answer on Question #55436 – Math – Calculus

what is integrals??? is it an operator or no??

Solution

An operator is a mapping between two function spaces.
The operation of integration is the inverse of differentiation.
For this reason, the term integral may also refer to the related notion of the antiderivative, a function *F* whose derivative is the given function *f*.
In this case, it is called an indefinite integral and it is written in the following way:

$$F(x) = \int f(x)dx$$

We can define the operator of indefinite integration on the spaces $C[0,1]$ and $L_1[0,1]$.

If function $f(x)$ is continuous on the interval $[a,b]$, then the function

$G(x) = \int_a^x f(t)dt$ **is differentiable and $G'(x) = f(x)$ for every x in the interval $[a,b]$.**

If function $f(x)$ is integrable with Lebesgue measure in $\mathbb{R}$, then the function $G(x) = \int_a^x f(t)dt$ is continuous. Moreover, if $f(x)$ is continuous at x, then $G'(x) = f(x)$.

Integral $F(x)$ is a linear operator, because

$$\int\big(f(x) + g(x)\big)dx = \int f(x)dx + \int g(x)dx$$

and $\int cf(x)dx = c\int f(x)dx$.

Integral $G(x)$ is also a linear operator, because

$$\int_a^x (f(t) + g(t))dt = \int_a^x f(t)dt + \int_a^x g(t)dt$$

and

$$\int_a^x cf(t)dt = c\int_a^x f(t)dt.$$

Thus, integral is an operator.