Answer on Question #55277 – Math – Trigonometry

What is the value of $sin^2(\alpha) + sin^2(\beta) + sin^2(\gamma)$? Why?

Solution

The direction angles α, β and γ are the angles that the vector makes with the positive *x*-, *y*- and *z*-axes, respectively.

If α, β and γ are the direction angles, then $cos^2(\alpha) + cos^2(\beta) + cos^2(\gamma) = 1$.

Compute

$$sin^2(\alpha) + sin^2(\beta) + sin^2(\gamma) = (1 - cos^2(\alpha)) + (1 - cos^2(\beta)) + (1 - cos^2(\gamma)) =$$

$$= 3 - \left(cos^2(\alpha) + cos^2(\beta) + cos^2(\gamma)\right) = 3 - 1 = 2$$

If α, β and γ are the angles of any triangle (where $\alpha + \beta + \gamma = 180°$ and each of $\alpha, \beta,$ and γ is greater than zero), then $cos^2(\alpha) + cos^2(\beta) + cos^2(\gamma) + 2cos(\alpha)cos(\beta)cos(\gamma) = 1$.

Compute

$$sin^2(\alpha) + sin^2(\beta) + sin^2(\gamma) = (1 - cos^2(\alpha)) + (1 - cos^2(\beta)) + (1 - cos^2(\gamma)) =$$

$$= 3 - \left(cos^2(\alpha) + cos^2(\beta) + cos^2(\gamma) + 2cos(\alpha)cos(\beta)cos(\gamma)\right) + 2cos(\alpha)cos(\beta)cos(\gamma) =$$

$$= 3 - 1 + 2cos(\alpha)cos(\beta)cos(\gamma) = 2 + 2cos(\alpha)cos(\beta)cos(\gamma).$$

Answer: $sin^2(\alpha) + sin^2(\beta) + sin^2(\gamma) = 2$.