Answer on Question #55151 – Math – Algebra

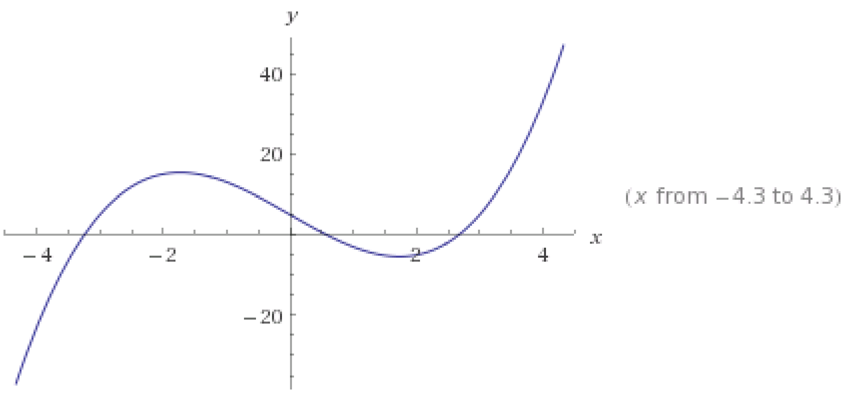
Question

Use the Intermediate Value Theorem to find intervals of length 1 which contain the real zeros of $f(x) = x^3 - 9x + 5$.

Solution

The first method is analytical with application of the Intermediate Value Theorem. First, just starting anywhere, $f(0) = 5 > 0$. Next, $f(1) = -3 < 0$. So, since $f(0) > 0$ and $f(1) < 0$, there is at least one root in $[0,1]$, by the Intermediate Value Theorem. Next, $f(2) = -5 < 0, f(3) = 5 > 0$. So, since $f(2) < 0$ and $f(3) > 0$, by the Intermediate Value Theorem there is a root in $[2,3]$. Now if we somehow imagine that there is a negative root as well, then we try -1: $f(-1) = 13 > 0$. So we know nothing about roots in$[-1,0]$. But continue: $f(-2) = 15 > 0, f(-3) = 5 > 0$ and still no new conclusion. Continue: $f(-4) = -23 < 0$. So, since $f(-3) > 0$ and $f(-4) < 0$, by the Intermediate Value Theorem there is a third root in the interval $[-4, -3]$.
Then, by the Intermediate Value Theorem, there are the zeros in the intervals $[0,1]$, $[2,3]$ and $[-4, -3]$.

The second method is graphical. We search for points, where the graph crosses the x-axis.
Plot of the function $f(x) = x^3 - 9x + 5$ is given below.



There are the zeros in the intervals $[0,1]$, $[2,3]$ and $[-4, -3]$.