ANSWER ON QUESTION #55014 – Math – Linear Algebra

Solve the linear equation 2x+y-3z= 5,3x-2y-2z= 5, and 5x-3y-z= 16.

Solution

We shall solve the given system

$$\begin{cases} 2x + y - 3z = 5, \\ 3x - 2y - 2z = 5, \\ 5x - 3y - z = 16 \end{cases}$$

with help of Cramer's rule.

The determinants of this system are the following:

$$\Delta = \begin{vmatrix} 2 & 1 & -3 \\ 3 & -2 & -2 \\ 5 & -3 & -1 \end{vmatrix} = 2\begin{vmatrix} -2 & -2 \\ -3 & -1 \end{vmatrix} - \begin{vmatrix} 3 & -2 \\ 5 & -1 \end{vmatrix} - 3\begin{vmatrix} 3 & -2 \\ 5 & -3 \end{vmatrix} =$$

$$= 2[(-2)(-1) - (-3)(-2)] - [3(-1) - 5(-2)] - 3[3(-3) - 5(-2)] =$$

$$= 2(2-6) - (-3+10) - 3(-9+10) = -18 \neq 0$$

(hence, there exists a unique solution to the given system),

$$\Delta_x = \begin{vmatrix} 5 & 1 & -3 \\ 5 & -2 & -2 \\ 16 & -3 & -1 \end{vmatrix} = 5\begin{vmatrix} -2 & -2 \\ -3 & -1 \end{vmatrix} - \begin{vmatrix} 5 & -2 \\ 16 & -1 \end{vmatrix} - 3\begin{vmatrix} 5 & -2 \\ 16 & -3 \end{vmatrix} =$$

$$= 5[(-2)(-1) - (-3)(-2)] - [5(-1) - 16(-2)] - 3[5(-3) - 16(-2)] = 5(2-6) - (-5+32) - 3(-15+32) = -98,$$

$$\Delta_y = \begin{vmatrix} 2 & 5 & -3 \\ 3 & 5 & -2 \\ 5 & 16 & -1 \end{vmatrix} = 2\begin{vmatrix} 5 & -2 \\ 16 & -1 \end{vmatrix} - 5\begin{vmatrix} 3 & -2 \\ 5 & -1 \end{vmatrix} - 3\begin{vmatrix} 3 & 5 \\ 5 & 16 \end{vmatrix} =$$

$$= 2[5(-1) - 16(-2)] - 5[3(-1) - 5(-2)] - 3[3 \cdot 16 - 5 \cdot 5] = 2(-5+32) - 5(-3+10) - 3(48-25) = -50,$$

$$\Delta_z = \begin{vmatrix} 2 & 1 & 5 \\ 3 & -2 & 5 \\ 5 & -3 & 16 \end{vmatrix} = 2\begin{vmatrix} -2 & 5 \\ -3 & 16 \end{vmatrix} - \begin{vmatrix} 3 & 5 \\ 5 & 16 \end{vmatrix} + 5\begin{vmatrix} 3 & -2 \\ 5 & -3 \end{vmatrix} =$$

$$= 2[(-2)16 - (-3)5] - [3 \cdot 16 - 5 \cdot 5] + 5[3 \cdot (-3) - 5(-2)] = 2(-32+15) - (48-25) + 5(-9+10) = -52.$$

By Cramer's rule, the unique solution to the given system is given by

$$x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}, z = \frac{\Delta_z}{\Delta}.$$

Therefore,

$$x = \frac{98}{18} = \frac{49}{9}, y = \frac{50}{18} = \frac{25}{9}, z = \frac{52}{18} = \frac{26}{9}$$

Answer: $x = \frac{49}{9}, y = \frac{25}{9}, z = \frac{26}{9}$.