Answer on Question #54903 – Math – Calculus

Find the radius of convergence and interval of convergence

of the series

1. Sum of ((-1)^n)(x^n)/((1/n)^3) with n=1 ->infinite

2. Sum of ((-1)^n)(x^n)/(n^2) with n=1 ->infinite

3. Sum of (n^n)(x^n) with n=1 ->infinite

4. Sum of ((2x-1)^n)/(5(1/n)^n) with n=1 ->infinite

Solution

1. $\sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} (-1)^n \cdot x^n n^3$

The radius of convergence is $R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\left|\frac{(-1)^n n^3}{(-1)^{n+1}(n+1)^3}\right| = \lim_{n\to\infty}\left|\frac{(n)^3}{(n+1)^3}\right| = 1$.

The series converges if $-1 < x < 1$, and diverges if $x < -1$ or $x > 1$.

At the endpoint $x = -1$ we have $\sum_{n=1}^{\infty} n^3$, at the endpoint $x = 1$ we have $\sum_{n=1}^{\infty}(-1)^n n^3$. In both cases $(-1)^n n^3$ and n^3 do not converge to zero as $n \to \infty$, hence both series diverge.

If $x = \mp 1 \Rightarrow \sum_{n=1}^{\infty} a_n x^n$ is not convergent.

interval of convergence $x \in (-1;1)$.

2. $\sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot x^n}{n^2}$

The radius of convergence is $R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\left|\frac{\frac{(-1)^n}{n^2}}{\frac{(-1)^{n+1}}{(n+1)^2}}\right| = \lim_{n\to\infty}\left|\frac{(n+1)^2}{(n)^2}\right| = 1$,

If $x = \mp 1 \Rightarrow \sum_{n=1}^{\infty} a_n x^n$ is convergent. Series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ is absolutely convergent,

because $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent as the generalized harmonic series with $p = 2$.

interval of convergence $x \in [-1,1]$.

3. $\sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} x^n n^n$

The radius of convergence is

$$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\left|\frac{(-1)^n n^n}{(-1)^{n+1}(n+1)^{n+1}}\right| = \lim_{n\to\infty}\left|\frac{(n)^n}{(n+1)^{n+1}}\right| = \lim_{n\to\infty}\left|\frac{(n)^n}{(n+1)^n (n+1)}\right| = \lim_{n\to\infty}\left|\frac{1}{\left(\frac{n+1}{n}\right)^n (n+1)}\right| =$$

$$= \lim_{n\to\infty}\left|\frac{1}{\left(1+\frac{1}{n}\right)^n (n+1)}\right| = \lim_{n\to\infty}\left|\frac{1}{e(n+1)}\right| = 0$$

interval of convergence is $x = 0$ (one point).

4. $\sum_{n=1}^{\infty} a_n x^n = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot (2x-1)^n n^n}{5}$.

The radius of convergence is

$$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|=\lim_{n\to\infty}\left|\frac{\frac{(-1)^n\cdot n^n}{5}}{\frac{(-1)^{n+1}\cdot(n+1)^{n+1}}{5}}\right|=\lim_{n\to\infty}\left|\frac{(n)^n}{(n+1)^{n+1}}\right|=\lim_{n\to\infty}\left|\frac{(n)^n}{(n+1)^n(n+1)}\right|=\lim_{n\to\infty}\left|\frac{1}{\left(\frac{n+1}{n}\right)^n(n+1)}\right|=,$$

$$=\lim_{n\to\infty}\left|\frac{1}{\left(1+\frac{1}{n}\right)^n(n+1)}\right|=\lim_{n\to\infty}\left|\frac{1}{e(n+1)}\right|=0$$

$$2x-1=0\Rightarrow x=0.5$$

interval of convergence is $x=0.5$ (one point).