Answer on Question #54902 – Math – Calculus

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

1. $\sum_{n=1}^{\infty} \frac{(-1)^{(n-1)(n)}}{n^2+4}$

2. $\sum_{n=1}^{\infty} \frac{n!}{100^n}$

3. $\sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}}$

4. $\sum_{n=1}^{\infty} \frac{3-cos(n)}{n^{\frac{2}{3}}-2}$

Solution

1.

$$\sum_{n=1}^{\infty} \frac{(-1)^{(n-1)(n)}}{n^2+4} \le \sum_{n=1}^{\infty} \left| \frac{(-1)^{(n-1)(n)}}{n^2+4} \right| = \sum_{n=1}^{\infty} \frac{1}{n^2+4} \le \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

This is an example of series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, which is convergent if p>1. In our case p=2. Thus the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent and $\sum_{n=1}^{\infty} \frac{(-1)^{(n-1)(n)}}{n^2+4}$ is absolutely convergent.

2.

Using the Ratio Test,

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{(n+1)!}{100^{n+1}}}{\frac{n!}{100^n}} \right| = \lim_{n\to\infty} \left| \frac{n+1}{100} \right| = \infty.$$

Thus, the series is divergent.

3.

$$\sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}} \le \sum_{n=1}^{\infty} \left| \frac{n^{10}}{(-10)^{n+1}} \right| = \frac{1}{10} \sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$$

Using the Ratio Test,

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{\frac{(n+1)^{10}}{10^{n+1}}}{\frac{n^{10}}{10^n}} \right| = \lim_{n\to\infty} \left| \frac{\left(1+\frac{1}{n}\right)^{10}}{10} \right| = \frac{1}{10}.$$

Therefore, since $\frac{1}{10} < 1$, the Ratio Test says that the series $\sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$ converges.

That's why, by the Comparison Test, the series $\sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}}$ is absolutely convergent.

4.

$$\sum_{n=1}^{\infty}\frac{3-cos(n)}{n^{\frac{2}{3}}-2} > \sum_{n=1}^{\infty}\frac{3-1}{n^{\frac{2}{3}}} = 2\sum_{n=1}^{\infty}\frac{1}{n^{\frac{2}{3}}}.$$

This is an example of series $\sum_{n=1}^{\infty}\frac{1}{n^p}$, which is convergent if p>1 and divergent if p≤1

In our case $p=\frac{2}{3}$. Thus, the series $2\sum_{n=1}^{\infty}\frac{1}{n^{\frac{2}{3}}}$ is divergent.

So, by the direct comparison test, the series $\sum_{n=1}^{\infty}\frac{3-cos(n)}{n^{\frac{2}{3}}-2}$ is divergent too.