Answer on Question #54901 – Math – Calculus

Question: Test the series for convergence or divergence

1.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+4)}$$

Solution: By the Alternating Series Test,

$$\lim_{n\to\infty} \frac{1}{\ln(n+4)} = 0$$

and for any n the inequality $\frac{1}{\ln(n+4)} > \frac{1}{\ln((n+1)+4)}$ holds true, because $\ln n$ is increasing.

It means that the series is convergent.

Answer: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+4)}$ is convergent

2.

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n^3+2}}$$

Solution: By the Alternating Series Test,

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^3+2}} = \lim_{n\to\infty} \frac{1}{\sqrt{n+\frac{2}{n^2}}} = 0$$

and

$$\frac{n}{\sqrt{n^3+2}} > \frac{n+1}{\sqrt{(n+1)^3+2}},$$

because

$$\frac{1}{\sqrt{n+\frac{2}{n^2}}} > \frac{1}{\sqrt{n+1+\frac{2}{(n+1)^2}}}$$

$$\sqrt{n+1+\frac{2}{(n+1)^2}} > \sqrt{n+\frac{2}{n^2}}$$

$$n+1+\frac{2}{(n+1)^2} > n+\frac{2}{n^2}$$

$1 > \frac{2}{n^2} - \frac{2}{(n+1)^2}$ holds true for $n \geq 2$.

Hence the series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{\sqrt{n^3+2}}$ converges.

Answer: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+4)}$ is convergent

3.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{2n+3}$$

Solution: By the Alternating Series Test,

$$\lim_{n\to\infty} \frac{\sqrt{n}}{2n+3} = \lim_{n\to\infty} \frac{1}{2\sqrt{n} + \frac{3}{\sqrt{n}}} = 0$$

and

$$\frac{\sqrt{n}}{2n+3} > \frac{\sqrt{n+1}}{2(n+1)+3}$$

Solve the last inequality. Since both sides are positive, squared them and consider ratio

$$\frac{n}{4n^2+6n+9} : \frac{n+1}{4n^2+10n+25} = \frac{4n^3+10n^2+25n}{4n^3+6n^2+9n+4n^2+6n+9} = \frac{4n^3+10n^2+15n+10n}{4n^3+10n^2+15n+9} > 1$$

for any n because $10n > 9$

Inequality is valid and the series is convergent.

Answer: $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{2n+3}$ is convergent

4.

$$\sum_{n=1}^{\infty} \frac{n \cos n\pi}{2^n}$$

Solution: Since $\cos n\pi = (-1)^n$ we have series

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{2^n}$$

Using the Alternating Series Test obtain

$\lim_{n\to\infty} \frac{n}{2^n} = 0$ since by the l'Hospital's rule $\lim_{x\to\infty} \frac{x}{2^x} = \lim_{x\to\infty} \frac{1}{2^x \ln 2} = 0$

and $\frac{n}{2^n} > \frac{n+1}{2^{n+1}}$ for n>1, because

$2^{n+1} n > 2^n n + 2^n$

$2^n n > 2^n$

$n > 1.$

The series $\sum_{n=1}^{\infty} \frac{n \cos n\pi}{2^n}$ converges.

Answer: $\sum_{n=1}^{\infty} \frac{n \cos n\pi}{2^n}$ is convergent.