Answer on Question #54900 – Math – Calculus

Determine whether the series converges or diverges

1. Sum of (2k-1)(k^2-1)/(k+1)((k^2+4)^2) with k=1 ->infinite

Solution

Method 1

The integral

$$\int_1^\infty \frac{(2x-1)(x^2-1)dx}{(x+1)(x^2+4)^2} = \int_1^\infty \frac{-3x-7}{(x^2+4)^2}dx + \int_1^\infty \frac{2}{x^2+4}dx = -3\int_1^\infty \frac{x}{(x^2+4)^2}dx - 7\int_1^\infty \frac{1}{(x^2+4)^2}dx + 2\int_1^\infty \frac{1}{x^2+4}dx = \lim_{t\to\infty}\left[-3\left(-\frac{1}{2(t^2+4)}\right) - \frac{7}{16}\left(\frac{2t}{t^2+4} + tan^{-1}\left(\frac{t}{2}\right)\right) + \frac{2}{2}tan^{-1}\left(\frac{t}{2}\right)\right] - 3\left(-\frac{1}{2(1^2+4)}\right) - \frac{7}{16}\left(\frac{2}{1^2+4} + tan^{-1}\left(\frac{1}{2}\right)\right) + \frac{2}{2}tan^{-1}\left(\frac{1}{2}\right) = \frac{1}{32}(9\pi - 4 - -18\tan^{-1}(\frac{1}{2})) \approx 0.49777$$

converges, hence the series

$$\sum_{k=1}^\infty \frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}$$

converges according to the integral test.

Method 2

Function $f(x) = \frac{(2x-1)(x^2-1)}{(x+1)(x^2+4)^2}$ is equivalent to $g(x) = \frac{2x^3}{x^5} = \frac{2}{x^2}$ as $x \to \infty$, that is,

$$\lim_{x\to\infty}\frac{f(x)}{g(x)} = 1.$$

The integral

$$\int_1^\infty \frac{2}{x^2}dx = \lim_{t\to\infty}\left(-\frac{2}{t} + 2\right) = 2$$

converges, hence the integral

$$\int_1^\infty \frac{(2x-1)(x^2-1)}{(x+1)(x^2+4)^2}dx$$

converges.

Thus, the series

$\sum_{k=1}^{\infty} \frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}$

converges according to the integral test.

2. Sum of sqrt(n)/(n-1) with n=2 ->infinite

Solution

Method 1

The integral

$\int_2^{\infty} \frac{\sqrt{x}dx}{x-1} = |y = \sqrt{x}, x = y^2| = \int_{\sqrt{2}}^{\infty} \frac{2y\cdot ydy}{y^2-1} = \int_{\sqrt{2}}^{\infty} \left(2 + \frac{1}{y-1} - \frac{1}{y+1}\right) dy = \lim_{t\to\infty} \left(2t + ln\frac{t-1}{t+1} - \right.$
$\left. -2\sqrt{2} - \ln\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = \infty$

diverges hence the series diverges according to the integral test.

Method 2

Function $f(x) = \frac{\sqrt{x}}{x-1}$ is equivalent to $g(x) = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$ as $x \to \infty$, that is,

$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1.$

The integral

$\int_1^{\infty} \frac{1}{\sqrt{x}} dx = \lim_{t\to\infty} \left(2\sqrt{t} - 2\right) = +\infty$

diverges, hence the integral

$$\int_1^{\infty} \frac{\sqrt{x}}{x-1} dx$$

diverges.

Thus, the series

$\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$

diverges according to the integral test.

Method 3

Because

$$\frac{\sqrt{n}}{n-1} > \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}}$$

and the series

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}} = -1 + \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

diverges as the generalized harmonic series with $\frac{1}{2} < 1$, the series $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$ diverges according to comparison test.

3. Sum of 1/(2n+3) with n=1 ->infinite

Solution

Method 1

The integral $\int_1^{\infty} \frac{dx}{2x+3} = \frac{1}{2} \lim_{t\to\infty} (\ln(2t+3) - \ln 5) = \infty$ diverges, hence this series diverges according to the integral test.

Method 2

Function $f(x) = \frac{1}{2x+3}$ is equivalent to $g(x) = \frac{1}{2x}$ as $x \to \infty$, that is,

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 1.$$

The integral

$$\int_1^{\infty} \frac{1}{2x} dx = \frac{1}{2} \lim_{t\to\infty} (ln|t| - 0) = +\infty$$

diverges, hence the integral

$$\int_1^{\infty} \frac{1}{2x+3} dx$$

diverges.

Thus, the series

$$\sum_{n=1}^{\infty} \frac{1}{2n+3}$$

diverges according to the integral test.

4. Sum of (n+2)/((n+1)^3) with n=3 ->infinite

Solution

The integral

$$\int_3^{\infty}\frac{(x+2)dx}{(x+1)^3}=\int_3^{\infty}\frac{1}{(x+1)^2}dx+\int_3^{\infty}\frac{1}{(x+1)^3}dx=\lim_{t\to\infty}\left(-\frac{1}{t+1}-\frac{1}{2(t+1)^2}+\frac{1}{3+1}+\frac{1}{2(3+1)^2}\right)=\frac{9}{32}$$

converges, hence the series $\sum_{n=1}^{\infty}\frac{n+2}{(n+1)^3}$ converges according to the integral test.