Answer on Question #54899 – Math – Calculus

Determine whether the series is convergent or divergent

1. 1+1/2.sqrt(2) +1/3.sqrt(3) +1/4.sqrt(4) +...

Solution

The series

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$$

converges, because it is the generalized harmonic series and 3/2>1.

2. Sum of n^2/(n^3+1) with n=1 ->infinite

Solution

The integral

$$\int_1^{\infty} \frac{x^2 dx}{x^3+1} = \frac{1}{3} \lim_{t\to\infty} [\ln(t^3+1) - \ln 2] = \infty$$

diverges, hence the series diverges according to the integral test.

3. Sum of (3n-4)/(n^2-2n) with n=3 ->infinite

Solution

The integral

$$\int_3^{\infty} \frac{(3\mathrm{x}-4)dx}{x^2-2x} = \int_3^{\infty} \frac{3x-4}{x(x-2)} dx = \int_3^{\infty} \left(\frac{2}{x} + \frac{1}{x-2}\right) dx = \lim_{t\to\infty} [2ln|t| + ln|t-2| - 2ln3 - 2ln1] = \lim_{t\to\infty} [\ln(\mathrm{t}^2(t-2)) - \ln 9] = \infty$$

diverges, hence the series diverges according to the integral test.

4. Sum of 1/(n^2 +6n+13) with n=1 ->infinite

Solution

The integral

$$\int_1^{\infty} \frac{dx}{x^2+6x+13} = \int_1^{\infty} \frac{dx}{(x+3)^2+4} = \lim_{t\to\infty} \frac{1}{2}\left(tan^{-1}\frac{t+3}{2} - tan^{-1}\frac{1+3}{2}\right) = \frac{1}{2}\left(\frac{\pi}{2} - \tan^{-1} 2\right)$$

converges, hence the series converges according to the integral test.