Answer on Question #54898 – Math – Calculus

1. Calculate the first eight terms of the sequence of partial sums correct to four decimal places. Does it appear that the series is convergent or divergent?

Sum of ((-1)^(n-1))/(n!) with n=1 ->infinite

Solution

First eight partial sums are equal to 1., 0.5, 0.6667, 0.625, 0.6333, 0.6319, 0.6321, 0.6321.

It appears that the series is *convergent* to a number close to 0.63.

2. Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

a) 4+3+9/4+27/16+...

Solution

The geometric series can be transformed as follows:

$$4 + 3 + \frac{9}{4} + \frac{27}{16} + \cdots = 4\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n = 4 \cdot \frac{1}{1 - {}^{3}\!/_{4}} = 16$$

The series *converges*, its *sum* is 16.

b) Sum of (1)/((sqrt(2))^n) with n=0 ->infinite

Solution

The geometric series can be transformed as follows:

$$\sum_{n=0}^{\infty}\frac{1}{\left(\sqrt{2}\right)^n} = \frac{1}{1 - {}^{1}\!/_{\sqrt{2}}} = \frac{\sqrt{2}}{\sqrt{2} - 1} = \frac{\sqrt{2}\left(\sqrt{2} + 1\right)}{\left(\sqrt{2} - 1\right)\left(\sqrt{2} + 1\right)} = \frac{2 + \sqrt{2}}{2 - 1} = 2 + \sqrt{2}$$

The series *converges*, its *sum* is $2 + \sqrt{2} \approx 3.41.$

c) Sum of (e^n)/(3^(n-1)) with n=1 ->infinite

Solution

The geometric series can be transformed as follows:

$$\sum_{n=1}^{\infty}\frac{e^n}{3^{n-1}} = 3\sum_{n=1}^{\infty}\left(\frac{e}{3}\right)^n = 3(\sum_{n=0}^{\infty}\left(\frac{e}{3}\right)^n - 1) = 3\left(\frac{1}{1-{}^{e}/_{3}} - 1\right) = 3\left(\frac{3}{3-e} - 1\right)$$
$$= 3\cdot\frac{3-3+e}{3-e} = \frac{3e}{3-e}$$

The series *converges*, its *sum* is $\frac{3e}{3-e} \approx 28.95.$