**Answer on Question #54874 – Math – Statistics and Probability**

A random sample of size 64 has been drawn from a population with standard deviation 20. The mean of the sample is 80.

**i)** Calculate 95% confidence limits for the population mean.

**ii)** How does the width of the confidence interval changes if the sample size is 256 instead?

**Solution**

In the given task we have the following data: $\bar{x} = 80, n = 64, s = 20$.

**i)**

In order to compute the confidence interval for μ, we will need the t multiplier and the standard error:

$$SE = \frac{s}{\sqrt{n}}$$

Substitute the given values into the formula noted above:

$$SE = \frac{20}{\sqrt{64}} = 2.5$$

Then, we determine the critical value. The critical value is a factor used to compute the margin of error. For this example, we'll express the critical value as a t-score.

We compute alpha:

$$\alpha = 1 - \frac{95}{100} = 0.05$$

Find the critical probability:

$$p = 1 - \frac{\alpha}{2} = 1 - \frac{0.05}{2} = 0.975$$

Then, we can calculate the degrees of freedom:

$$df = n - 1 = 64 - 1 = 63$$

The critical value is the t-score having 63 degrees of freedom and a cumulative probability equal to 0.975 will be equal to 1.998 (From the Table of Critical values of Student's

t-distribution with v degrees of freedom).

The formula for the confidence interval for a population mean is

$$\bar{x} \pm t_{1-\alpha,v} \frac{s}{\sqrt{n}}$$

Thus, our confidence interval for μ is

$$80 \pm 1.998 \cdot \frac{20}{\sqrt{64}}$$

Simplify the obtained expression:

$$80 \pm 4.995$$

$$75.005 \leq \mu \leq 84.995$$

We are 95% confident that the population mean is between 75.005 and 84.995. The upper limit is 84.995 and the lower limit is 75.005.

We might also have expressed the critical value as a z score. Because the sample size is fairly large, a z-score analysis produces a similar result - a critical value equal to 1.96.

Now, we determine the margin of error:

$$\text{ME} = 1.96 \cdot 2.5 = 4.9$$

Finally, we specify the confidence interval:

$$80 \pm 4.9$$

$$75.1 \leq \mu \leq 84.9$$

We are 95% confident that the population mean is between 75.1 and 84.9.

**ii) How does the width of the confidence interval changes if the sample size is 256 instead?**

First, we note some important facts about the confidence interval. $t_{1-\alpha,v}\frac{s}{\sqrt{n}}$ is the margin of error; other things being equal, the margin of error of a confidence interval increases as the sample size $n$ decreases. Thus, when the sample size decreases, the length of the confidence interval will become bigger. When the sample size increases, the confidence interval will be smaller. However, it will become bigger as the confidence level increases. Therefore, we cannot conclude how the confidence interval will be in this question, since we don't have enough information to determine whether the change in sample size or the confidence level is more influential here.

The sampling distribution of a statistic is the distribution of values taken by the statistic in all possible samples of the same size from the same population. The mean of the sampling distribution of $\bar{x}$ is $\mu$, the population mean.

Now, we change the value of the sample size and find the standard error. Substitute the given values into the formula noted above:

$$\text{SE} = \frac{20}{\sqrt{256}} = 1.25$$

Thus, taking into account the calculation indicated above, our confidence interval for μ will be equal:

$$80 \pm 1.96 \cdot \frac{20}{\sqrt{256}}$$

Simplify the obtained expression:

$$80 \pm 2.45$$

$$77.55 \leq \mu \leq 82.45$$

We are 95% confident that the population mean is between 77.55 and 82.45, if the sample size is 256. Based on the obtained results, the width of the confidence interval will be smaller, if the sample size is 256 instead of 64.